

# Projecting stand attributes of regrowth ash eucalypts sampled in forest inventory

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## Summary

An empirical stand growth model was developed for natural regrowth of mountain ash (*Eucalyptus regnans*) and alpine ash (*Eucalyptus delegatensis*) in Central Victoria. The model can be used to project the stand attributes sampled in Victoria's Statewide Forest Resource Inventory (SFRI) for sustainable yield forecasts. Because a generalised stand table projection approach was adopted, a wide range of stand structures can be projected including both even-aged, unimodal diameter distribution stands and uneven-aged, multi-modal diameter distribution stands. Validation using repeat measurements from permanent plots indicates that for projection intervals less than 20 y, mean bias of projected yields was generally within  $\pm 8\%$  for stocking per hectare and  $\pm 0.5\%$  for basal area per hectare for the two species. Projected future stand tables were evaluated against observed stand tables using the two-sample Kolmogorov–Smirnov test. The null hypothesis that the predicted and observed diameter distributions arise from the same distribution could not be rejected for more than 84% of cases tested. It was concluded that the model is reliable for projecting SFRI data into future.

*Keywords:* mathematical models; stand characteristics; stand tables; forest inventories; growth; sustainability; forest management; *Eucalyptus delegatensis*; *Eucalyptus regnans*

## Introduction

Native ash eucalypt forests, dominated by mountain ash (*Eucalyptus regnans* F.Muell.) or alpine ash (*Eucalyptus delegatensis* R.T.Bak.) in the Central and Eastern Highlands of Victoria, are the State's fastest growing and most commercially important hardwood forests. About half of the sustainable timber yield from public native forests comes from these forests (Incoll and Dignan 1997). Most ash stands originated from 1939 fires. Their early development was characterised by high initial stocking and rapid height growth. The initial stocking as high as 490 000 to 2 400 000 stems ha<sup>-1</sup> thinned exponentially to 2000 stems ha<sup>-1</sup> at 20 y age (Ashton 1976).

These regrowth forests are currently managed for clearfelling at a nominal rotation age of 80 y, followed by slash burning and

seeding for regeneration. Although mono-specific, even-aged stands predominate, some ash stands contain trees of different ages (e.g. regrowth, advance growth and overwood), and some also contain a mixture of species. Ashton (1976) observed that even-aged ash stands result from high-intensity crown fires that kill both overstorey and understorey. However, low-intensity ground fires, where the understorey may be killed but the overstorey survives or is only partially killed, can produce dual or multi-aged stand structures. For example, about 38% of mountain ash forests in the Maroondah and O'Shannassy catchments are dual or multi-aged stands (Ough and Ross 1992).

Victoria's Department of Sustainability and Environment (DSE; formerly Natural Resources and Environment, DNRE) is implementing a Statewide Forest Resource Inventory (SFRI) on the State's 3.47 million ha of public native forests. The primary objective of the SFRI is to provide forest managers with a comprehensive set of forest resource data for making informed and consistent sustainable yield forecasts, and decisions on forest land-use planning and resource allocation (DNRE 1997a). At every SFRI sampling point, two sets of data are collected: detailed tree and stand attributes, and stem defect parameters for determining net merchantable volume of the sampled stands. These data are then analysed and modelled with other site variables, including altitude and latitude, to estimate current net merchantable volumes for major forest classes in regional forest management units, called Forest Management Areas (FMAs). The current areas and estimates of volume of forest stands from the SFRI, together with future yield data predicted by forest growth models, are essential inputs to the Integrated Forest Planning System used by DSE to forecast sustainable yields (DNRE 1997b).

The growth model STANDSIM, previously used to predict future yields of ash eucalypts in Victoria, is generally not appropriate for the SFRI. That model was developed using repeat measurements of permanent growth plots (PGPs) in mono-specific, even-aged ash stands on relatively good sites (Opie 1972; Incoll 1983). The SFRI, however, covers a wide range of sites and stand conditions, including mixed-species and mixed-age stands. Furthermore, the current four-grade hardwood utilisation standards were introduced after STANDSIM had been built.

Additional procedures must be developed to permit the deduction of defect volumes if the STANDSIM model is to be used for SFRI yield projections.

The objective of the work described in this paper was to develop an alternative growth model to STANDSIM for projecting the stand attributes sampled in the SFRI to estimate future yields of regrowth mountain ash and alpine ash forests.

## Data description

Data for the study include repeat measurements from 160 PGPs and estimated net merchantable volumes for 1061 sample trees from SFRI field plots in the Tambo and Central Gippsland FMAs.

### PGP data

PGPs were selected from a database including over 800 permanent hardwood plots managed by DSE. Most plots were originally established for spacing or commercial thinning trials. Criteria used to select PGPs from the database were that each plot had to be dominated by mountain ash or alpine ash that had developed from naturally regenerated stands, and that it had either not been thinned (48 control plots) or had been thinned from below (112 plots). Selected PGPs were from 25 localities across the distributed areas of ash forests in Victoria (Fig. 1) and had been measured 4–19 times at intervals varying from 1 to 6 y. The measurement periods of these plots varied from 6 to 67 y. The plots are rectangular in shape and their sizes vary from 0.04 to 0.45 ha. Frequency distributions of the 160 selected PGPs by species, site index and age classes are shown in Table 1 and their descriptive statistics at the time of first measurement are shown in Table 2.

Along with the usual measures of tree and stand attributes taken in a PGP system, data from log assessments were also available. A log assessment involves grading potential logs in sample trees according to the State's four-grade utilisation standards for hardwood sawlogs, denoted as A, B, C and D grade logs. 'A' grade logs are relatively high quality with few stem defects, while 'D'

**Table 1.** Frequency distributions of 160 permanent growth plots by species, site index and stand age classes at plot establishment

Species	Age (y)	Site index (m)			
		22.5	27.5	32.5	37.5
Mountain ash	10	2	5	5	2
	30	15	9	6	2
	50	2	2	1	
Alpine ash	10		2	10	7
	30	4	18	42	9
	50		10	6	
	110			1	

Note: Site index is stand top height (m) at the 20 y reference age.

**Table 2.** Descriptive statistics of 160 permanent growth plots based on the first measurement

Variable	Number of plots	Mean	Minimum	Maximum
Age (y)	160	27.4	4.0	100.0
Site index (m)	160	30.3	20.3	39.1
Stocking (stems ha <sup>-1</sup> )	160	841.0	45.1	11185.2
Basal area (m <sup>2</sup> ha <sup>-1</sup> )	160	29.1	0.9	72.3
QMD (cm)	160	33.0	5.4	90.5

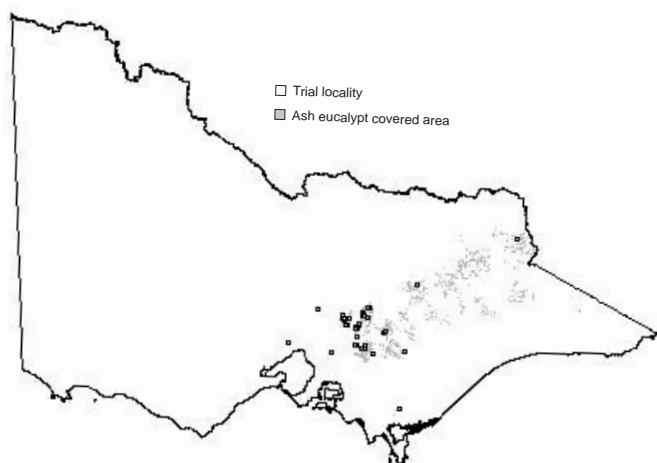
Note: QMD = quadratic mean diameter.

grade logs are relatively low quality with relatively extensive stem defects. Measurements taken in log assessments include the diameter outside bark and height from the ground, for each sawlog or piece of residual roundwood identified.

Information on stand stocking and basal area from the PGPs was used to derive the stand-level projection functions for future stocking and basal area per hectare. Log assessment data were used to develop the regression models for predicting total log heights and the probability of a tree having some sawlog volume of grade D or better (referred to as D+ sawlog in later discussion).

### SFRI data

The SFRI uses a model-based sampling scheme to select sample points from forest stands based on a pre-specified sampling frame (Hamilton *et al.* 1999). A variable-radius plot with a basal area factor of 3 is established at each sampling point to estimate stand basal area and stocking per hectare. All 'in' trees with a diameter at breast height (1.3 m above ground) over bark ( $d_{1.3}$ ) of 20 cm or greater are measured to estimate the diameter distribution of merchantable trees in the stand. A sub-sample of three trees is selected from all 'in' trees using the modified point-list method based on the cumulative estimated merchantable heights (Wood and Wiant 1992). These trees are then measured using a laser dendrometer to collect detailed data on the stem profile and external defect indicators based on a measurement process described as treemapping (Hamilton *et al.* 1999). Top height, defined as mean height of 62 largest diameter trees per hectare, percentage crown cover, and species composition of the stand are also recorded. A sub-set of the treemap sample trees in each



**Figure 1.** Localities of 160 permanent growth plots in ash eucalypt forests

FMA is felled to obtain data of the type, extent and incidence of internal stem defects (pipe). These data are analysed to estimate net volumes of D+ sawlog and residual roundwood of sample trees, and then net D+ sawlog and net total merchantable volumes per hectare of the sample stands.

In this study, the SFRI estimates of net D+ sawlog volume and net total merchantable volumes from 1061 treemap sample trees of mountain ash or alpine ash collected in the Tambo and Central Gippsland FMAs were used to develop volume equations for predicting net merchantable volume of standing trees. The descriptive statistics of these 1061 sample trees are in Table 3.

### Modelling methods

Forest stand growth and yield models vary in structural complexity and output detail, and from whole-stand to individual-tree resolution. Depending on the availability of data and application requirements, different approaches can be used to develop stand growth models (Clutter *et al.* 1983; Vanclay 1994).

In this study, the generalised stand table projection approach suggested by Nepal and Somers (1992) was adopted. This modelling approach uses stand-level yield functions for predicting future stocking and basal area per hectare, and a computational algorithm to derive future stand tables. Nepal and Somers indicated that their approach is appropriate for modelling both even-aged, unimodal diameter distribution stands and uneven-aged, multi-modal diameter distribution stands. When initial diameter distributions are unimodal, the approach provides estimated future stand tables that have a precision similar to that of the diameter-distribution-based growth models using parameter recovery procedures (e.g. Cao and Burkhart 1984). However, if the initial diameter distributions are multi-modal, the approach gives better estimates of future stand tables than those predicted

by the diameter distribution-based growth models. In the following subsections, methods used to develop the component functions of the alternative growth model are described.

### Two-stage modelling for projecting stand stocking

To project current stand conditions using the generalised stand-table projection approach suggested by Nepal and Somers (1992), stand stocking at a specified projection age must be predicted. In this study, the stand-level difference equation was selected as the base model for stand-stocking projection. A difference equation for stand-stocking projection takes the following form:

$$N_2 = f(N_1, T_2, T_1), \tag{1}$$

where  $N_2$  is total surviving trees per hectare (stocking) at age  $T_2$ ,  $N_1$  is total surviving trees per hectare at age  $T_1$ , and the difference between  $T_1$  and  $T_2$  is the projection interval.

Difference equations have been widely used for projecting stand stocking in forestry because of their desirable properties, including consistency, projection path-invariance, and the asymptotic limit of stocking approaching zero when the projected stand ages become great (Clutter *et al.* 1983). However, Woollons (1998) indicated that a conflict exists between the mathematical characteristics of difference equations for stocking projection and the data used to estimate their parameters. Mathematically, no matter how short the projection interval ( $T_2 - T_1$ ) specified, a difference equation will always predict some loss of stocking.

The most common data for estimating stocking projection functions are long-term measurements from a PGP system. In even-aged stands, it is possible that there will be no mortality over many measurements of PGPs, when the measurement interval

**Table 3.** Descriptive statistics of the 1061 treemap sample trees from the Tambo and Central Gippsland SFRI

Species	Variable	Number of trees	Mean	Minimum	Maximum
Mountain ash	$d_{1.3}$ (cm)	325	72.9	21.3	223.4
	Top point height (m)	325	31.4	3.0	55.0
	Cumulative log length (m)	325	32.0	5.9	54.0
	Net D+ sawlog volume (m <sup>3</sup> )	325	2.9	0.0	10.8
	Net total merchantable volume (m <sup>3</sup> )	325	3.9	0.1	17.5
Alpine ash	$d_{1.3}$ (cm)	736	61.6	20.6	165.6
	Top point height (m)	736	22.8	3.0	45.0
	Cumulative log length (m)	736	22.6	0.0	51.6
	Net D+ sawlog volume (m <sup>3</sup> )	736	1.4	0.0	8.1
	Net total merchantable volume (m <sup>3</sup> )	736	2.1	0.0	10.9

Notes: Top point height = a measure of merchantable tree height; cumulative log length = total length of all identified sawlog and residual roundwood logs in a tree; net total merchantable volume = sum of net D+ sawlog and residual roundwood volumes.

is short (e.g. 5 y or less). When nonlinear least squares are used to estimate the parameters of a difference equation, measurements having no mortality make no contribution to estimating the model parameters. However, if these measurements are ignored, bias may be introduced into the predictions when the estimated models are applied to predict stand stocking. Besides this problem, irregular measurement intervals of the PGP data will also introduce bias when predicting stand stocking.

In the PGP data, 8% of the measurements in the unthinned PGPs and 61% of the measurements in the thinned PGPs did not include any mortality. To solve these problems, a two-stage modelling approach suggested by Woollons (1998) was adopted. First, all measurements with or without mortality were used to develop a logistic regression equation to predict the probability of mortality occurrence in a stand for a one-year period. The best difference equation for projecting stand stocking was then selected and applied, using only the measurements that included mortality. Only measurements from the PGPs with an interval of 1, 2 and 3 y were used when estimating the model. Data with a measurement interval of 2 or 3 y were transformed into 1-y measures using linear interpolation before the model was applied.

### Projecting stand basal area

Stand basal area per hectare at a specified projection age is also required when applying the algorithm suggested by Nepal and Somers (1992). Many functional forms have been suggested in the forestry literature for stand basal area projection (e.g. Clutter *et al.* 1983; Murphy and Farrar 1988). Clutter and Jones (1980) developed the following difference equation for basal-area projection:

$$G_2 = G_1^{(T_1/T_2)^{c_1}} \exp \left\{ c_2 \left[ \left( 1 - \frac{T_1}{T_2} \right)^{c_1} \right] \right\}, \quad (2)$$

where  $G_1$  and  $G_2$  are stand basal area per hectare at ages  $T_1$  and  $T_2$  years ( $T_1 \leq T_2$ ),  $\exp$  is the base of natural logarithm; and  $c_1$  and  $c_2$  are parameters to be estimated. Pienaar *et al.* (1985) showed that the yield function mathematically compatible to (2) is:

$$G = \theta^{(1/T)^{c_1}} \exp(c_2), \quad (3)$$

where  $\theta$  is an integral parameter; and  $G$  is stand basal area per hectare at age  $T$ .

To project the basal area growth for both thinned and unthinned stands, Pienaar *et al.* (1985) modified (2) by incorporating a measure of thinning intensity ( $X$ ). The modified model was:

$$G_2 = G_1^{(T_1/T_2)^{c_0+c_1X}} \exp \left\{ c_2 \left[ \left( 1 - \frac{T_1}{T_2} \right)^{c_0+c_1X} \right] \right\}, \quad (4)$$

where  $X = \text{TPA}_t / \text{TPA}_a$ , and  $\text{TPA}_t$  and  $\text{TPA}_a$  are the trees per hectare removed in thinning, and remaining after thinning respectively.

Although (4) has the advantage of accounting for thinning effects, it requires knowledge of the number of trees prior to thinning.

Many PGPs used in this study were established after thinning, and pre-thinning conditions were not known. Therefore (4) could not be applied directly. Instead, parameter prediction (Clutter *et al.* 1983) was used to relate parameter  $c_2$  of equation (2) to a thinning indicator, a site index estimate, and other stand attributes. The site index of a stand was defined as the mean height of the 62 largest diameter trees per hectare (i.e. stand top height) at a reference age of 20 y. In the SFRI project, the site indices (SI) of stands were estimated from stand ages (AGE) and observed top height (TOPH) using a site index equation. For mountain ash and alpine ash, the site index equation was (Campbell *et al.* 1979):

$$\text{TOPH} = 2 \times \text{SI} \times (1 - 10^{-0.015 \times \text{AGE}}). \quad (5)$$

### Estimating tree volume

To derive the estimates of net merchantable volumes per hectare from projected frequency distributions of tree diameters (i.e. future stand tables), tree volume prediction equations are required. Sustainable yield rates in Victoria are defined as net volumes of D+ sawlog and residual logs that allow for the deduction of defect volumes (DNRE 1997b). To meet this requirement, the log assessment data from PGPs and the estimated net D+ sawlog volumes and net total merchantable volumes from the 1061 sample trees described earlier were used as the database to develop models of net merchantable volume per tree.

Four component equations were developed to estimate net merchantable volumes of mountain ash and alpine ash trees:

- i) a logistic regression equation for estimating the probability of a tree having some D+ sawlog volume;
- ii) a nonlinear regression equation for estimating total merchantable height of trees;
- iii) a volume equation for estimating net volume of D+ sawlog per tree; and
- iv) a volume equation for estimating the total net merchantable volume per tree (i.e. the sum of net D+ sawlog volume and residual log volume).

### Model identification

#### Models for projecting stand stocking

To develop the logistic regression equation for estimating the probability of occurrence of mortality, a binary response variable was created by coding each PGP measurement as 1 if mortality occurred, and 0 if no mortality occurred. The logit transformation of the response variable was then related to tree and stand attributes and external stem defect indicators using a linear regression model (Hosmer and Lemeshow 1989). The LOGISTIC procedure of the Statistical Analysis System (SAS) package (SAS Inc. 1994) was used for parameter estimation. The best combination of predictor variables was selected based on the Akaike Information Criterion (AIC) and Schwartz Criterion (SC). The significance of each candidate model estimated was determined using the  $-2 \log$  likelihood and score test statistics. Based on goodness-of-fit testing and plotting of residual deviances, the best logistic regression equation selected for both mountain ash and alpine ash was:

$$p = [1 + \exp(-(\beta_0 + \beta_1 \ln(T_1) + \beta_2 \ln(N_1) + \beta_3 \ln(SI) + \beta_4 \text{TREAT} \times T_1))]^{-1}, \quad (6)$$

where  $p$  is the estimated probability of mortality occurrence in a stand for a 1-y period, TREAT is an indicator variable with TREAT = 1 for thinned stands and TREAT = 0 for unthinned stands, and  $\beta_0, \beta_1, \beta_2, \beta_3$  and  $\beta_4$  are parameters to be estimated. Species was also coded as an indicator variable and tested in model estimation, but it was not statistically significant. The parameter estimates, fitting and testing statistics for (6) are given in Table 4.

Several difference equations for stocking projection, described by Clutter *et al.* (1983), were examined using only stocking measurements of PGPs, including mortality. Nonlinear regression was used to estimate the parameters of each candidate projection function using the NLIN procedure of SAS. The following function was found to give the minimum error for predicting stand stocking and showed no serious bias in residual plots for both species of interest:

$$N_2 = \left( \frac{1}{\sqrt{N_1}} + \beta \times \left[ \left( \frac{T_2}{100} \right)^2 - \left( \frac{T_1}{100} \right)^2 \right] \right)^{-2}, \quad (7)$$

where  $\beta$  is the parameter to be estimated. Equation (7) was fitted separately for mountain ash and alpine ash (Table 5). As indicated earlier, stand stocking projection functions were estimated for a 1-y period. When the desired projection period is longer than 1-y, the future stand stocking will be obtained in an iterative fashion by setting the predicted  $N_2$  as  $N_1$  to initiate the next projection until the specified projection age is reached.

Equations (6) and (7) form a two-stage system for projecting the current stand stocking. The estimate of future stand stocking is obtained as:

$$\hat{N}_{\text{Adj}} = N_1 - \hat{p} \times (N_1 - \hat{N}_2), \quad (8)$$

where  $\hat{N}_{\text{Adj}}$  is the adjusted stand stocking per hectare at age  $T_2$ ;  $\hat{N}_2$  is the stand stocking predicted from (7), and  $\hat{p}$  is the probability of mortality occurrence in a 1-y period predicted from (6).

**Table 4.** Parameter estimates, fit and test statistics of the logistic regression equation for predicting the mortality probability of both mountain ash and alpine ash (equation (6))

Parameter	Estimate	Standard error	Fit and test statistics	
$\beta_0$	-24.1593	1.4187	AIC	3657.1
$\beta_1$	2.7475	0.1357	SC	3688.3
$\beta_2$	1.7632	0.0743	-2 Log L	1630.1 ( $P < 0.0001$ )
$\beta_3$	1.6558	0.3134	Score	1374.9 ( $P < 0.0001$ )
$\beta_4$	-0.0342	0.0029	Goodness-of-Fit	63.4 ( $P < 0.0001$ )

Notes: AIC = Akaike Information Criterion; SC = Schwartz Criterion; -2 Log L =  $-2 \times \log$  likelihood test statistic; Score = score test statistic; and Goodness-of-Fit = goodness-of-fit test statistic.

**Table 5.** Parameter estimates and fit statistics for stocking projection equations (equation (7))

Species	Parameter/fit statistic	Estimate for unthinned stands	Estimate for thinned stands
Mountain ash	$N$	681	651
	$\beta$	0.2756 (0.011)	0.07272 (0.003)
	$R^2$	0.98	0.97
Alpine ash	RMSE	140.6	8.3
	$N$	259	263
	$\beta$	0.2322 (0.009)	0.09564 (0.005)
	$R^2$	0.96	0.98
	RMSE	78.0	6.9

Notes: Standard errors are shown in brackets;  $N$  = number of PGP measurements; and RMSE = root mean squared error.

**Models for projecting stand basal area**

To develop the regression equation for predicting future stand basal area per hectare, the Model procedure of SAS was used to estimate the parameters of equation (2) and the specified  $c_2$  function, simultaneously. The data used for parameter estimation were similar to those used to estimate the stand mortality functions. A large number of functional forms for relating  $c_2$  were examined and the best model was:

$$G_2 = G_1^{(T_1/T_2)^{c_1}} \exp \left\{ c_2 \left[ \left( 1 - \frac{T_1}{T_2} \right)^{c_1} \right] \right\} \quad (9)$$

where

$$c_2 = \frac{a_0}{c_1} + \frac{a_1}{c_1} \times SI + \frac{a_2}{c_1} \times \text{TREAT} + \frac{a_3}{c_1} \times \text{SPCODE},$$

where SPCODE = 1 for mountain ash and 0 for alpine ash, and the other variables are as defined previously. The parameter estimates and fitting statistics for (9) are given in Table 6.

For a given initial basal area per hectare, (9) is used to project future stand basal area per hectare in a recursive fashion from the initial to the final age specified. In general, as a stand ages, the predicted basal area per hectare increases and reaches an asymptotic value ( $e^{c_2}$ ) at advanced ages. Predicted basal area per hectare for stands on good sites or stands that have been thinned reaches an asymptotic value sooner than for stands on poor sites

**Table 6.** Parameter estimates and fit statistics for basal area projection equations for both mountain ash and alpine ash (equation (9))

Parameter	Estimate	Standard error	Fit statistics	
$a_0$	2.5508	0.2018	$N$	3749
$a_1$	0.0150	0.0037	RMSE	1.18
$a_2$	-0.1977	0.0316		
$a_3$	-0.1433	0.0344	$R^2$	0.99
$c_1$	0.6089	0.0396	Adj	

Notes:  $R^2$  = adjusted  $R^2$ ; RMSE = root mean squared error;  $N$  = number of PGP measurements.

or stands that have not been thinned. Predicted basal areas per hectare for mountain ash and alpine ash stands are similar, but mountain ash stands generally have a higher asymptotic value.

## Models for estimating net merchantable volume of trees

### Models for estimating probability of D+ sawlog occurrence

To predict the probability of a tree having some D+ sawlog volume, logistic regression analysis was performed using the data from PGP log assessment and SFRI treemapping. Each sample tree was coded as 1 if there was some D+ sawlog volume and 0 if there was no D+ sawlog volume. Statistical procedures similar to those used to estimate the stand stocking projection functions were applied to develop the logistic regression equations for predicting the probability of D+ sawlog occurrence. The best equation found for mountain ash was:

$$\hat{p}(\text{D+ sawlog}) = [1 + \exp[-(-31.1928 + 8.5150 \times \ln(d_{1.3}) - 0.0330 \times G)]]^{-1}, \quad (10)$$

where  $\hat{p}(\text{D+ sawlog})$  is the predicted probability of a tree having D+ sawlog volumes,  $\ln$  is natural logarithmic transformation, and  $G$  is stand basal area per hectare. The best equation for alpine ash was:

$$\hat{p}(\text{D+ sawlog}) = [1 + \exp[-(-25.6802 + 3.5635 \times \sqrt{d_{1.3}} - 0.0111 \times G)]]^{-1}, \quad (11)$$

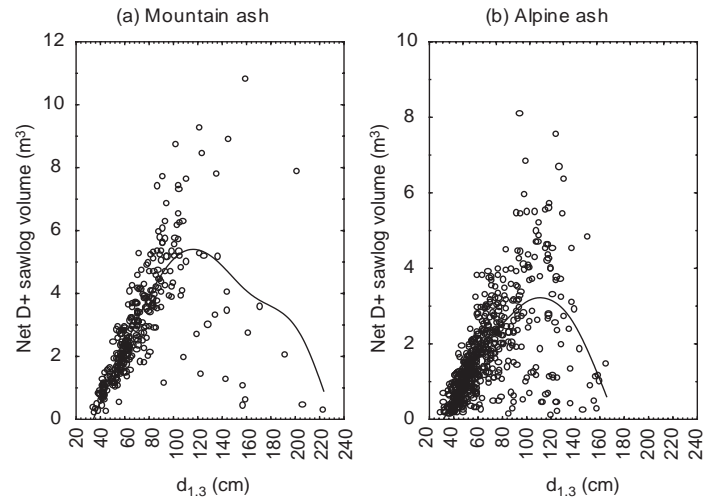
Sample trees were originally coded using a binary indicator variable to separate trees from unthinned or thinned PGPs in both estimated models. However, the estimated coefficients for the indicator variable in both equations were not significantly different from zero, and were dropped from the equations. Therefore, the equations do not differentiate between thinned and unthinned stands.

### Models for estimating net D+ sawlog volume

To develop an equation for estimating net D+ sawlog volume per tree, the estimates of net D+ sawlog volumes of treemap trees from the SFRI were plotted against  $d_{1.3}$ . Various smoothing approaches were then used to study the general trend of mean net D+ volumes. It was found that as tree  $d_{1.3}$  increases, the mean net D+ sawlog volume per tree from the SFRI increases until a 'threshold  $d_{1.3}$ ' value is reached. For the data used, the threshold  $d_{1.3}$  value was between 100 to 120 cm (Fig. 2). The mean net D+ sawlog volumes gradually decreased with increasing  $d_{1.3}$  after this point, due to an increase in internal stem defects in large-diameter trees. To model this general trend, the following base function described by Ratkowsky (1990) was used:

$$Y = a \times d_{1.3}^b \times e^{-c \times d_{1.3}}, \quad (12)$$

where  $Y$  = net D+ sawlog volume of a sample tree,  $e$  is the base of natural logarithm (i.e. 2.7183); and  $a$ ,  $b$ ,  $c$  are parameters to be estimated.



**Figure 2.** Plots of net D+ sawlog volume versus  $d_{1.3}$  for sample trees collected in the Tambo and Central Gippsland SFRI (curves are least squares smoothed)

Various modifications to (12) were fitted to data for the net D+ volume of the treemap sample trees using nonlinear least squares. The best equation was:

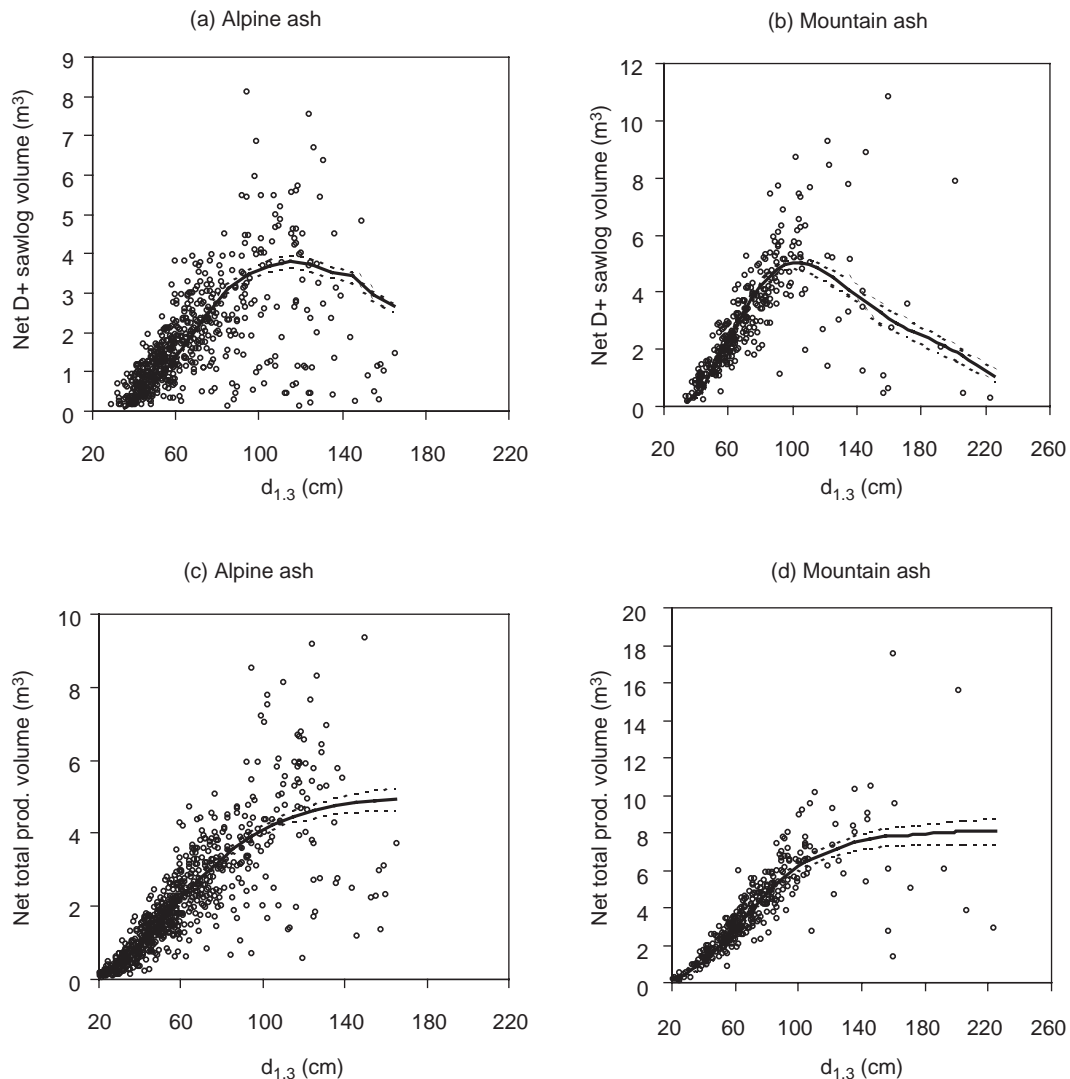
$$V_{\text{D+ sawlog}} = A_0 \times (d_{1.3} - 30)^{A_1} \times (\text{HT}_{\text{Log}} - 6)^{A_2} \times e^{-A_3 \times (d_{1.3} - 30)}, \quad (13)$$

where  $V_{\text{D+ sawlog}}$  is the estimate of net D+ sawlog volume per tree,  $\text{HT}_{\text{Log}}$  is the total merchantable height (m) of a tree, the constants 30 and 6 are the minimum values for  $d_{1.3}$  and  $\text{HT}_{\text{Log}}$ , respectively, of the treemap sample trees with a non-zero net D+ sawlog volume, and  $A_0$ ,  $A_1$ ,  $A_2$  and  $A_3$  are parameters to be estimated. The parameter estimates and fitting statistics of (13) are given in Table 7. The estimated models and 95% confidence intervals for the conditional mean of response variables were plotted with overlying observed net D+ sawlog volumes of treemap sample trees (Fig. 3(a), 3(b)).

**Table 7.** Parameter estimates and fit statistics of the volume equation for estimating net D+ sawlog volume per tree (equation (13))

Species	Parameter/ fit statistic	Estimate
Mountain ash	$N$	310
	$A_0$	0.0008152 (0.0004)
	$A_1$	1.9856 (0.1456)
	$A_2$	0.5501 (0.0691)
	$A_3$	0.02207 (0.0022)
	$R^2$	0.68
	RMSE	1.04
Alpine ash	$N$	589
	$A_0$	0.0008091 (0.0003)
	$A_1$	1.7999 (0.1345)
	$A_2$	0.7282 (0.0523)
	$A_3$	0.02228 (0.0025)
	$R^2$	0.62
	RMSE	0.81

Note: Numbers in brackets are standard errors of the parameter estimates; RMSE = root mean square error.



**Figure 3.** Smoothed trends of predicted net D+ sawlog and net total merchantable volumes (solid lines), their 95% confidence intervals (dashed lines) and observed volumes (dots) of sample trees

In practical situations, merchantable height is unknown for a given tree unless it is felled and the logs are graded. Therefore, another prediction equation was required to estimate the merchantable height,  $HT_{Log}$ . The Chapman–Richard function (Clutter *et al.* 1983) was selected for this purpose based on the plots of observed merchantable heights against tree  $d_{1.3}$  values. The log assessment data from the PGP’s and the treemap tree data were fitted separately using the NLIN procedure of SAS and then evaluated for predictive ability and biological justification. The best equation for mountain ash was:

$$\text{est. } HT_{Log} = 0.75447 \times SI \times [1 - e^{-0.03773 \times d_{1.3}}]^{3.90397}, \quad (14)$$

where est.  $HT_{Log}$  is the estimated merchantable height (m) of a tree. Values for  $R^2$  and root mean squared error (RMSE) in the mountain ash model were 0.42 and 4.56 m, respectively. The best equation for alpine ash was:

$$\text{est. } HT_{Log} = 0.65476 \times SI \times [1 - e^{-0.08023 \times d_{1.3}}]^{19.75711}, \quad (15)$$

with an  $R^2$  and RMSE of 0.22 and 3.54 m, respectively. In applying (14) and (15), the estimate of merchantable height is set to 0 if

$d_{1.3}$  is less than or equal to 35 cm for mountain ash or 40 cm for alpine ash.

### Model for estimating net merchantable volume

To develop the regression equation for estimating net merchantable volume per tree, the estimated net merchantable volumes of treemap trees were plotted against  $d_{1.3}$  values. A sigmoidal trend was found. To model this relationship, the Chapman–Richard function was used:

$$V_{TOT \text{ Mer.}} = A \times [1 - e^{-B \times d_{1.3}}]^C, \quad (16)$$

where  $A$ ,  $B$  and  $C$  are parameters to be estimated. The parameter estimates and fitting statistics for (16) are given in Table 8. Residual roundwood volume of a tree was estimated as the difference between the predicted net total merchantable volume and D+ sawlog volume per tree. The estimated models and 95% confidence bounds for the conditional mean of response variables were plotted with overlying observed net total merchantable volumes of treemap sample trees (Fig. 3(c), 3(d)).

**Table 8.** Parameter estimates and fit statistics of the volume equation for estimating net total merchantable volume per tree (equation (16))

Species	Parameter/ fit statistic	Estimate
Mountain ash	$N$	327
	$A$	8.13470 (0.3545)
	$B$	0.03143 (0.0033)
	$C$	6.09734 (1.611)
	$R^2$	0.99
	RMSE	1.27
Alpine ash	$N$	739
	$A$	5.07601 (0.1931)
	$B$	0.03252 (0.0029)
	$C$	5.4202 (0.7631)
	$R^2$	0.99
	RMSE	0.92

Note: Numbers in brackets are standard errors of the parameter estimates; RMSE = root mean square error.

### Yield projection algorithm

The algorithm suggested by Nepal and Somers (1992) was extended to develop a size-class distribution-based growth model. The stand attributes used to initiate a yield projection were:

- i) current stand age ( $T_0$ );
- ii) stand age to be projected ( $T_p$ );
- iii) species code (0 for mountain ash; 1 for alpine ash);
- iv) thinning history indicator (1 for thinned; 0 for unthinned stands);
- v) site index estimate (SI); and
- vi) observed frequency distribution of tree diameters (i.e. current stand table), which contains the stems per hectare by 2-cm  $d_{1,3}$  classes (denoted as  $n_i$  and  $i = 1, \dots, k$ , where  $k$  is the number of diameter classes).

Other stand attributes associated with the initial stand attributes are the stocking per hectare ( $N_0$ ), basal area per hectare ( $G_0$ ) and the minimum and the maximum  $d_{1,3}$  values (MinD and MaxD).

The procedures for projecting the current stand conditions for one year into the future (i.e. from  $T_0$  to  $T_0+1$  years) are as follows (see Fig. 4 for flowchart of the procedures).

1. Initiate the projection by setting the initial age  $T_1$  to be the current age  $T_0$  years, the projected age  $T_2$  to be ( $T_0+1$ ) years, and initial stocking ( $N_1$ ) to be the current stocking ( $N_0$ ), and the initial basal area per hectare ( $G_1$ ) to be the current basal area per hectare ( $G_0$ ).
2. Predict the stocking per hectare ( $N_2$ ) and basal area per hectare ( $G_2$ ) at  $T_2$  using the stocking and basal area projection functions developed.
3. Estimate the values at age  $T_2$  for the midpoints of each diameter class included in the current stand table. To do this, use the following procedures adopted from Nepal and Somers (1992):
  - (a) Assume that tree diameter distributions at both initial age  $T_1$  and projected age  $T_2$  resemble a three-parameter

Weibull probability density function, and the following relationship exists (Bailey 1980):

$$D_2 = a_2 + b_2 \times \left( \frac{D_1 - a_2}{b_1} \right)^{c_1/c_2}, \quad (17)$$

where  $D_1$  and  $D_2$  are the midpoints of  $i$ th diameter class at  $T_1$  and  $T_2$  years respectively, and  $a_i$ ,  $b_i$  and  $c_i$  ( $i = 1$  or 2) are the location, shape and scale parameters at  $T_1$  and  $T_2$  years, respectively.

- (b) Estimate parameters  $b$  and  $c$  at ages  $T_1$  and  $T_2$  from the observed or predicted stand attributes at their corresponding ages using a parameter recovery method (i.e. Cao *et al.* 1982); the location parameter  $a$  is assumed to remain fixed at MinD.
  - (c) Apply (17) to the midpoints of each diameter class and MaxD in the current stand table. This will give the midpoints of diameter classes for the stand table at projected age  $T_2$ .
4. Update stems per hectare for each diameter class in the current stand table to  $T_2$  years, using the following procedures:
    - (a) First set stems per hectare in each diameter class at  $T_2$  years to equal those at the initial age  $T_1$ .
    - (b) Then adjust stems per hectare in different diameter classes to equate the stand stocking and basal area per hectare predicted in Step 2 (i.e.  $\hat{N}_2$  and  $\hat{G}_2$ ). This adjustment can be mathematically expressed as the following two constraints:

$$\sum_{i=1}^k P_i \times n_i = \hat{N}_2, \quad (18)$$

$$\sum_{i=1}^k g \times P_i \times n_i \times D_i^2 = \hat{G}_2, \quad (19)$$

where  $D_i$  is the midpoint of the  $i$ th diameter class,  $n_i$  is the unadjusted stems per hectare of the  $i$ th diameter class,  $g$  is the factor for converting squared diameter to basal area (i.e. 0.00007854 in metric units),  $P_i$  is the percent adjustment required for the stems per hectare in the  $i$ th diameter class, and the other variables are as defined previously.

- (c) Nepal and Somers (1992) determined  $P_i$  using the following function:

$$P_i = \alpha_0 \times e^{\alpha_1 \times D_i}. \quad (20)$$

Parameter  $\alpha_1$  in (20) can be numerically determined from (21) using a Secant search method (Burden and Faires 1989),

$$\sum_{i=1}^k \frac{g \times (e^{\alpha_1 \times D_i}) \times n_i \times D_i^2 \times \hat{N}_2}{\sum_k (e^{\alpha_1 \times D_i}) \times n_i} = \hat{G}_2, \quad (21)$$

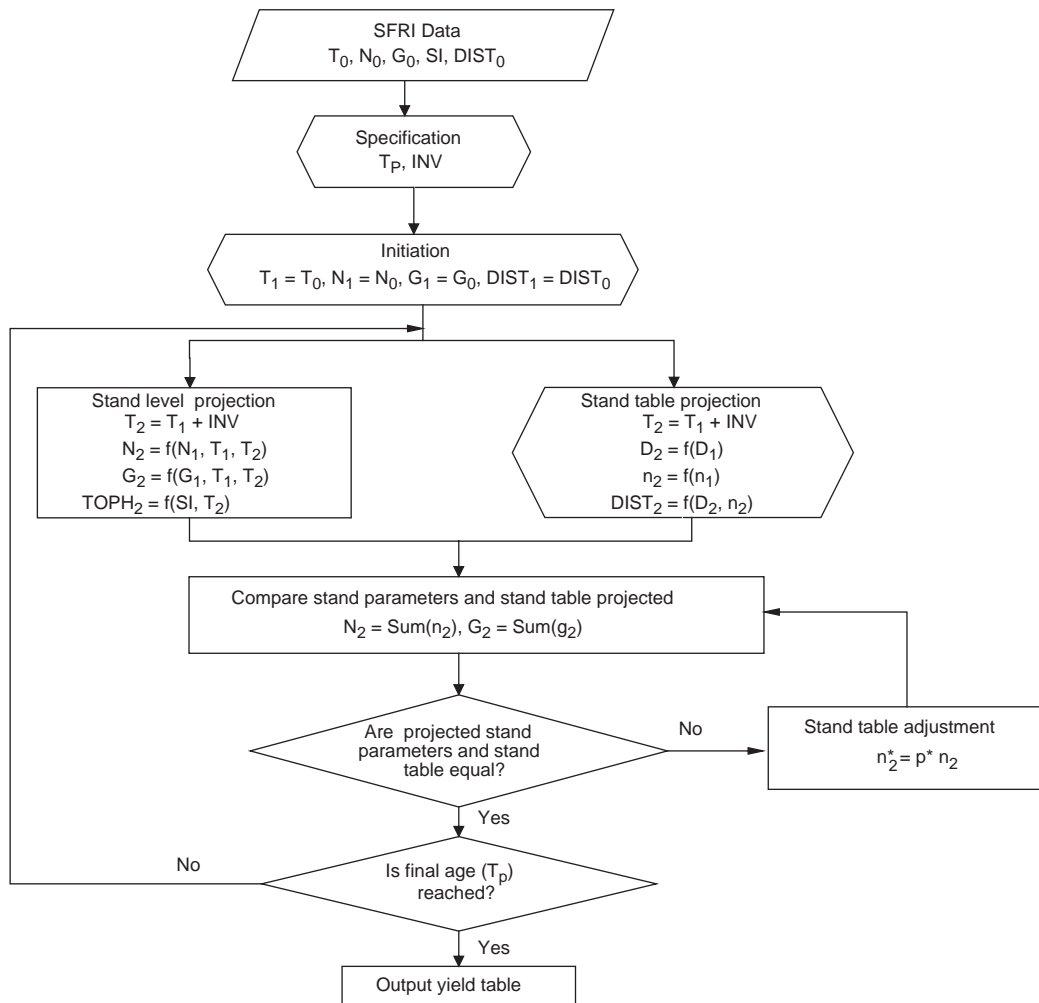


Figure 4. Flowchart illustrating yield projection process of the developed model

and  $\alpha_0$  in (20) is calculated after  $\alpha_1$  has been found:

$$\alpha_0 = \frac{\hat{N}_2}{\sum_k (e^{\alpha_1 \times D_i}) \times n_i} \quad (22)$$

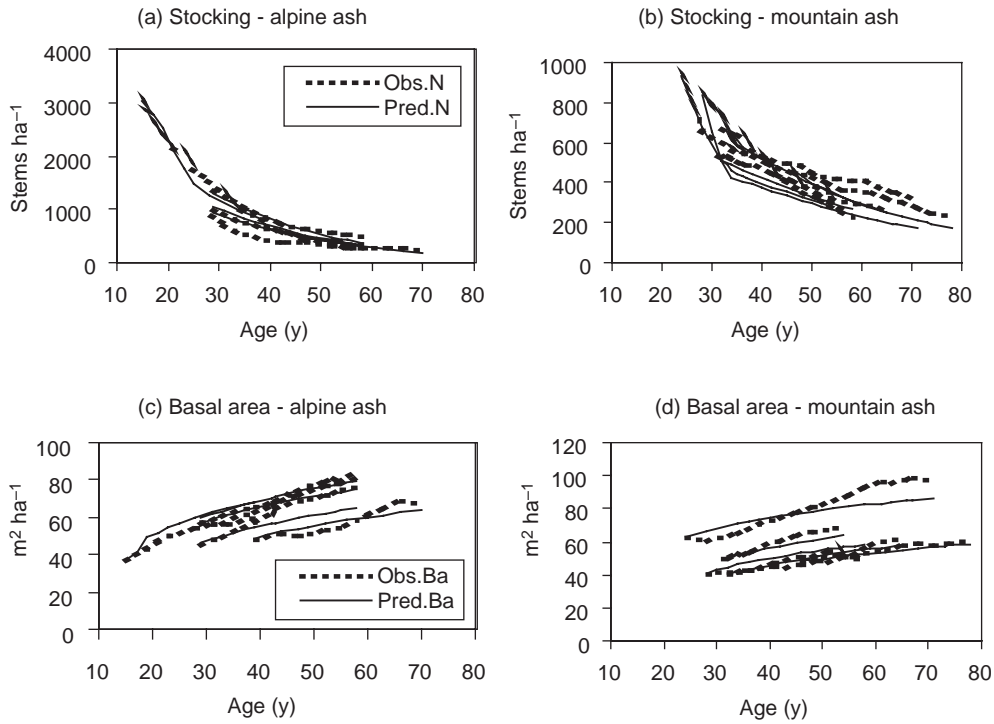
5. Apply the prediction equations for tree merchantable volumes described earlier to the midpoints of each diameter class in the updated stand table at  $T_2$  years.
6. Aggregate the estimates of net D+ sawlog and total net merchantable volumes of individual trees to obtain the net D+ sawlog and total net merchantable volumes of the stand at age  $T_2$ .
7. Reset the initial age  $T_1$  as  $T_2$  years and the projected age  $T_2$  as  $(T_2 + 1)$  years and repeat Steps 2 to 6.
8. Repeat Steps 2 to 7 until the projected age  $T_2$  equals the specified age  $T_p$ .

To assist in implementing the yield projections described, a computer program, ASHSTPS, was developed in C++.

## Evaluation of models

To evaluate the performance of the growth model, five PGPs were arbitrarily selected from the database for each of the two species. The frequency distributions of tree diameters at first measurement ranged from unimodal to irregular multi-modal in these selected PGPs. Data screens for the PGPs with a multi-modal diameter distribution showed that the stands might include regrowth, advanced growth trees and also some old (overwood) trees.

These PGPs were included in the data used to estimate the stand-level projection functions described earlier. However, only measurements with an interval of less than or equal to three years were used in developing the projection functions. All measurements, excluding the first measurement in each PGP, were compared to the future yields predicted from the growth model. Stand attributes and diameter distributions from the first measurement of each PGP were used as input to initiate the projections of future yields and stand tables.



**Figure 5.** Observed (Obs) versus projected (Pred) trajectories of stocking ( $N$ ) and basal area ( $Ba$ ) per hectare of the ten validation plots

Because most ash regrowth stands are unthinned, the evaluation focused on the unthinned PGPs. When plots were established in the selected PGPs, stand ages ranged from 13 to 36 y for alpine ash and from 21 to 43 y for mountain ash. Sixty-seven measurements were taken for alpine ash and 80 for mountain ash. For each selected PGP, stand stocking, basal area per hectare, site index estimate and observed frequency distribution in 2-cm diameter classes were used to initiate the projections using the computer program, ASHSTPS. Yield projections for each PGP were made to cover the measurement period. The average projection interval was 19 y for mountain ash and 16 y for alpine ash.

**Table 9.** Mean bias, mean percentage bias and associated standard errors of predicted stocking and basal area per hectare calculated for the ten validation plots

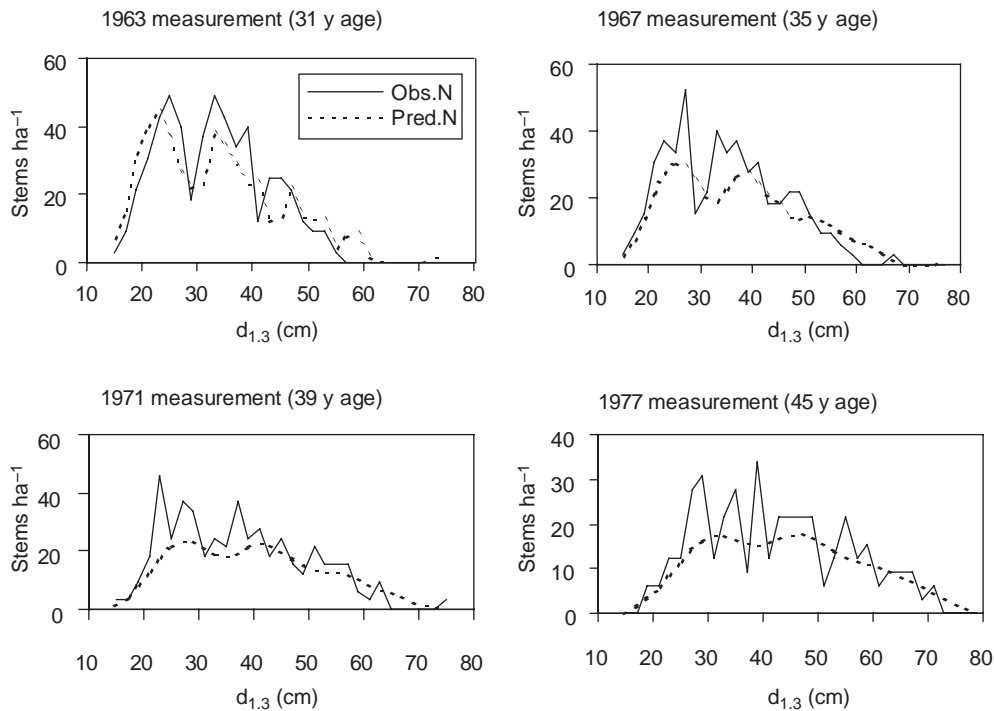
Species	Stand attribute	Number of projections	Average projection interval (y)	Mean bias	Mean percentage bias (%)
Mountain ash	Stocking (stems $ha^{-1}$ )	75	19	53.9 (17.4)	7.9 (1.7)
	Basal area ( $m^2 ha^{-1}$ )			0.9 (0.5)	0.4 (2.0)
Alpine ash	Stocking (stems $ha^{-1}$ )	62	16	-19.6 (17.3)	-4.1 (2.0)
	Basal area ( $m^2 ha^{-1}$ )			0.5 (0.6)	-0.2 (1.0)

Notes. Average projection interval is the average of total projection years of five PGPs; numbers in brackets are standard errors.

The accuracy of the predicted stocking and basal area per hectare were evaluated using the mean difference (prediction bias) and mean percentage difference (percentage prediction bias) between predicted and observed values. The accuracy of the predicted stand tables was evaluated using the two-sample Kolmogorov–Smirnov (K–S) test (Steel and Torrie 1980), a non-parametric procedure for testing the null hypothesis that the predicted and observed stand tables are from the same distribution. Stand net merchantable volume may be the most important stand attribute to evaluate, but treemapping for detailed stem profiles and external stem defects for the sample trees was not available. Therefore, the predicted net merchantable volumes were not evaluated.

Mean percentage bias of the predicted stocking per hectare was 7.9% for mountain ash, -4.1% for alpine ash and 2.5% for the two species combined (Table 9). Mean percentage bias of the predicted basal area per hectare was 0.4% for mountain ash, -0.2% for alpine ash, and 0.13% for the two species combined. For an average projection interval of 16 to 19 y, this result indicated that the developed model was making yield predictions with good accuracy. The observed stocking and basal area per hectare were also plotted against the predicted stocking and basal area per hectare for five validation plots and for mountain ash and alpine ash, respectively (Fig. 5). In general, the predicted stocking and basal area per hectare agreed well with the observed stocking and basal area per hectare across the entire range of projection periods for alpine ash. The predicted stocking for mountain ash appeared to be under-estimated, but no systematic bias was identified for the predicted basal area per hectare.

In the great majority of cases, the two-sample K–S tests could not reject the null hypothesis that the predicted and observed diameter frequency distributions (stand tables) arise from the same distribution. In other words, the predicted stand tables were not



**Figure 6.** Observed (Obs *N*) versus projected (Pred *N*) diameter distributions for four measures selected from the Erica thinning trial in Central Gippsland FMA

significantly different from the observed stand tables in 93% of cases for mountain ash and 84% for alpine ash. In addition, the predicted stand tables also maintained the multi-modal characteristic of the observed stand tables well in most projections and across the projection interval. An example is shown in Figure 6.

## Summary and conclusions

We have developed an alternative growth model for use in the SFRI, using modelling approaches that can account for both unimodal and multi-modal diameter distributions, based on the generalised stand table projection approach suggested by Nepal and Somers (1992). First, growth relationships between total stocking or basal area and other stand attributes were investigated using data from permanent growth plots. In general, the stocking of both species decreased rapidly at an early stage, but mortality became relatively small after 20–40 y.

The stocking vs. age relationship could be modelled using a two-stage projection system, the most useful predictors for future stocking being current stocking, site index, whether thinned or unthinned stands, and projection interval. Mean percentage bias for stand stocking predicted from the projection system was generally within  $\pm 8\%$  of the observed stocking for an average projection interval up to 19 y. Similarly, the relationship between basal area per hectare and age was analysed using nonlinear regression and a parameter prediction approach. Future basal area per hectare was correlated with current basal area per hectare, site index, species, and whether thinned or not. Mean percentage bias for stand basal area per hectare predicted from the model

was generally within  $\pm 0.5\%$  of observed stand basal area per hectare for an average projection interval up to 19 y.

Analysis of estimates of net product volume of sample trees from the SFRI in the Tambo and Central Gippsland forest management areas showed that, in general, net D+ sawlog volumes varied widely, due to the stochastic nature of stem defects. Trees with a  $d_{1,3}$  of less than 30 cm usually had no D+ sawlog volume. In larger trees, D+ sawlog volume increased as  $d_{1,3}$  increased up to a maximum  $d_{1,3}$  of 100–120 cm, then decreased with further increases in  $d_{1,3}$ .

Our equations for tree net merchantable volume, in conjunction with the estimated growth relationships of stocking and basal area, formed a generalised stand table projection growth model. In general, the resultant model is simple to apply, and uses full information collected in the SFRI to initiate yield projections. Furthermore, the yield procedures used combine the allocation of stand mortality and any growth adjustment necessary to ensure equality with the stand-level estimates. Because the observed stand table is updated directly using a diameter–growth relationship derived from the estimated diameter distribution at each projection period, the essential characteristics of the observed stand structures (e.g. multi-modal diameter distribution) can be preserved.

The results of model validation showed that projected stocking and basal area per hectare from the model generally agreed with their observed trajectories, and projection bias was acceptable for most applications. We conclude that the model is reliable for projecting the SFRI data into the future for forecasting sustainable yield.

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