

# A regionalised growth model for *Eucalyptus globulus* plantations in south-eastern Australia

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## Summary

A whole-stand distribution growth and yield model was developed for *Eucalyptus globulus* plantations in south-eastern Australia. The model was developed and parameterised using growth measurements from permanent sample or experimental plots established within plantation estates across the Gippsland, Central Victorian and Green Triangle regions. Regional variation of stand height and basal area growth, and mortality, was statistically tested using selected base models and indicator variables. Stand-level component models were constructed for projecting the stand heights, basal areas and stockings sampled in inventories into the future. Tree-level component models were developed for estimating the relationship between height and diameter, stem taper, and bark thickness at 1.3 m height from ground. To derive the diameter distributions of trees at any age, models were developed for predicting the 0th, 25th, 50th and 95th percentiles of cumulative diameter distributions. The Weibull distribution-based diameter distributions are estimated using the diameter percentiles and quadratic mean diameter predicted at the ages of interest (parameter recovery). In general, the developed models represent the mean growth of the *E. globulus* plantations in the three regions, and the observed and predicted yield trajectories agreed well for stand height, basal area and stocking. The accuracy of the developed stand models was evaluated using an independent data set. In general, for a prediction interval of 5 y or less, average bias in stand height, basal area and volume predictions is within  $\pm 1.0$  m,  $\pm 0.5$  m<sup>2</sup> ha<sup>-1</sup> and  $\pm 1.0$  m<sup>3</sup> ha<sup>-1</sup> respectively. The models are reliable for projecting the stand parameters sampled in inventories at ages 5 or 6 y to the end of the rotation at about age 10 y. The developed growth model is particularly useful for strategic planning and wood flow analysis.

**Keywords:** plantations; management; growth models; yield; regions; size; frequency distribution; planning; wood; flow; *Eucalyptus globulus*

## Introduction

Australia's total plantation area of 1.74 million ha (Mha) at December 2005 consisted of 0.74 Mha (43%) hardwoods and 0.99 Mha (57%) softwoods (Parsons *et al.* 2006). Between 1995 and 2005, there was a rapid expansion (0.54 Mha) of the total hardwood area, particularly in southern Australia where *Eucalyptus globulus* (Labill.), total 0.45 Mha, is mainly planted

in Western Australia, South Australia and Victoria, while *E. nitens* (Deane and Maiden), total 0.14 Mha, is usually planted in Tasmania. In Victoria and South Australia, industrial plantations of *E. globulus* have been established mainly in the Gippsland, Central Victorian and Green Triangle regions, with smaller-scale farm-forestry plantations in north-eastern Victoria.

Most *E. globulus* plantations are managed for pulpwood production on a nominal rotation of 10 y. Typical establishment silviculture includes soil cultivation by ripping, discing and/or mounding, pre-planting weed control, planting at a stocking of 1000–1200 trees ha<sup>-1</sup>, and fertiliser application. Generally *E. globulus* tolerates a wide range of site conditions (Beadle and Inions 1990), and has rapid growth (maximum mean annual increment up to 40 m<sup>3</sup> ha<sup>-1</sup>) on favourable sites (e.g. Inions 1992; Duncan *et al.* 2000). However, across the range of sites planted (about 600–1000+ mm annual rainfall), water availability is a primary limitation to growth.

Effective management of a plantation estate requires periodic inventories as a basis for assessing the yield potential (site productivity) of individual stands, and for predicting wood availability. Stand growth models for these purposes have been traditionally developed using repeated measurements from permanent sample plots established across the plantation population where the model will be applied (i.e. forest growth and yield models, Vanclay 1994). Many stand growth models have been developed around the world for different species and forest types (e.g. Clutter *et al.* 1983). However, there are few published models for *E. globulus* plantations in Australia, mainly because of the relatively short history of this industry, and because the plantations are owned and managed by numerous organisations, a situation that is not conducive to sharing data for model development.

The recent collaborative development of a 'Blue Gum (*E. globulus*) Plantation Management System' (Strandgard *et al.* 2005) provided an opportunity to develop *E. globulus* stand growth models for application across south-eastern Australia. The primary objective was to develop the growth models that can be used for projecting stand characteristics sampled in plantation inventories into the future for strategic planning and wood flow analysis. This paper presents the methods used and the developed models.

## Plantation regions and previous modelling

There are 15 regions in the Australian National Plantation Inventory (Parsons *et al.* 2006). *E. globulus* growth data from the Central Gippsland, and East Gippsland/Bombala (Victoria/New South Wales), Central Victorian and Green Triangle (Victoria/South Australia) regions were available for the present study. The Gippsland data were combined, and therefore the plantation regions for model development were Gippsland (GL), Central Victoria (CV) and Green Triangle (GT).

Forest growth and yield models vary in structural complexity and output detail, and from whole-stand to individual-tree resolution. Depending on the availability of data and application requirements, different methods may be used to develop forest growth models (Vanclay 1994). Previous growth modelling for *E. globulus* plantations in Australia, based on limited data, includes:

- A set of whole-stand growth models developed for south-eastern Australia (Wong *et al.* 2000). These models consist of three stand-level component models for predicting the temporal development of stand height, basal area and volume. Models were developed separately for *E. globulus*, *E. nitens* and *E. viminalis*, and a grouping of *E. botryoides*, *E. grandis* and *E. saligna* (Salignae series) using data from permanent plots in 12 plantation species trials in Gippsland and some data from south-eastern South Australia.
- A whole-stand growth model developed for south-western Western Australia (Inions 1992). Because of the lack of growth

data from permanent plots, the model was parameterised mainly using data from temporary plots. Component models were developed for estimating stand basal area and volume respectively. A stand height growth and site index model was also developed using height growth data derived from stem analysis of 87 trees sampled in 57 plots.

- An individual-tree, distance-independent growth model for northern Tasmania (Goodwin and Candy 1986). This model was developed using a limited data set from a single plantation. Component models for predicting tree height and diameter increments were derived. Diameter distributions of trees were modelled using a Beta probability stocking function, and stand mortality was approximated by the 3/2 self-thinning rule.

Most other published forest growth and yield models for *E. globulus* plantations have been developed in Portugal and Spain. These include stand-level models (e.g. Tome *et al.* 1995, 1997), tree-level growth models (e.g. Soares and Tome 2003) and a tree cohort model (i.e. Garcia and Ruiz 2003).

## Materials and methods

The main variables and mathematical expressions used in this paper are defined in Table 1.

### Data description

Three data sets were used for model development:

**Table 1.** Mathematical expressions, and abbreviations and definitions of variables

Symbol	Explanation or definition
dbhob	Tree diameter (cm) over bark at 1.3 m height
dbhub	Tree diameter (cm) under bark at 1.3 m height
$H$	Total tree height (m)
MDH	Dominant height or stand height (m), defined as the mean $H$ of the largest-diameter 100 trees $\text{ha}^{-1}$
$S$	Site index (m), defined as stand height at the reference age of 10 years
MDD	Dominant diameter (cm), defined as the mean dbhob of the largest-diameter 100 trees $\text{ha}^{-1}$
$D_q$	Quadratic mean diameter (dbhob, cm)
$N, BA, V$	Stand stocking (trees $\text{ha}^{-1}$ ), basal area ( $\text{m}^2 \text{ha}^{-1}$ ), volume ( $\text{m}^3 \text{ha}^{-1}$ ) at any age
MDH <sub>1</sub> and MDH <sub>2</sub>	Stand height (m) at stand age $T_1$ and $T_2$ years respectively
BA <sub>1</sub> and BA <sub>2</sub>	Stand basal area ( $\text{m}^2 \text{ha}^{-1}$ ) at stand age $T_1$ and $T_2$ years respectively
$N_1$ and $N_2$	Stand stocking (trees $\text{ha}^{-1}$ ) at stand age $T_1$ and $T_2$ years respectively
$D_0, D_{25}, D_{50}$ and $D_{95}$	0th, 25th, 50th and 95th percentiles (cm) of cumulative diameter (dbhob) distributions
$h$	Height (m) along tree stem above the ground ( $0 \leq h \leq H$ )
$d_h$	Stem diameter under bark (cm) at height $h$ (m)
DBT <sub>1.3</sub>	Double bark thickness at 1.3 m height from ground (cm)
ln	Natural logarithmic transformation
exp	The base of natural logarithms ( $\approx 2.71828$ )
$\beta_0$	Intercept parameter in a regression model
$\beta_0, \beta_1, \dots, \beta_k$	Slope parameters associated with each predictor variable in a regression model
$\hat{\beta}_0$	Estimated intercept parameter in a regression model
$\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_k$	Estimated slope parameters in a regression model
$n$	Total number of observations used in a regression
rmse	Root mean squared error of a regression model
$R_{\text{Adj}}^2$	Adjusted coefficient of determination of a regression model

1. Permanent plot data. This data set of repeated measurements of trees in permanent sample plots established by each collaborating organisation included:

- trial (series) information: seedlots, planting stocking, planting date and silvicultural treatments
- site and plot information: aspect, average slope, plot dimensions or area
- multiple measurements of trees: attributes recorded at each measurement include the identification number, status codes for health and mortality, diameter of all trees, and total height of a subset of trees.

The data set was used for the model selection and parameterisation in developing the stand-level component models, and also for the tree height–diameter relationship. Summary statistics for these data are presented in Table 2.

2. Stem taper data. Two sets of sectional stem diameter data from felled trees were available. The first set included 66 trees sampled across Victoria, and the second set included 189 trees sampled in central Gippsland. For each sample tree, stem diameter measurements (over and under bark) were recorded at 0.1 or 0.3 m (stump height), 1.3 m, 2.5 m, and thereafter at 1 or 2 m intervals along the stem, until a 2-cm top diameter was reached. These data were used to develop

the stem taper model and the bark thickness prediction model. Summary statistics for these data are presented in Table 3.

3. Additional permanent plot data. This data set was provided at a later stage of model development, and included information similar to that in the first set. In total, 579 plots in first-rotation *E. globulus* plantations on ex-agricultural sites in the Central Victoria region were available. Among these, 338 plots with three or more measurements were used. These data were reserved for evaluating the prediction accuracy of the developed models.

### Model structure and strategy for yield projection

A size-class growth model based on diameter distribution (Vanclay 1994) was used in this study because:

- size distribution information is essential for analysing the structure of plantations and making management decisions (e.g. estimating product assortment, or harvesting and transportation costs)
- the developed model should be flexible to permit analysis of alternative management options (e.g. spacing, thinning)
- available data including tree- and plot-level information should be used efficiently.

**Table 2.** Summary statistics<sup>1</sup> for permanent sample plot data used to develop the stand-level component models

Statistic	Region		
	Green Triangle	Central Victoria	Gippsland
Number of sites	52	20	14
Number of plots	138	192	338
Plot area (ha)	0.033 – 0.041 – 0.101	0.024 – 0.033 – 0.038	0.010 – 0.036 – 0.040
Total number of measurements	362	1049	949
Number of measurements per plot	2 – 2.8 – 8	4 – 6.3 – 9	1 – 3.4 – 7
Measurement interval (y)	1 – 1.3 – 3	1 – 1.1 – 2	1 – 2.3 – 6.4
Planting stocking (trees ha <sup>-1</sup> )	1000 – 1143 – 1301	960 – 1127 – 1300	900 – 1054 – 1236
Age (y)	1.0 – 3.2 – 12.0	2.0 – 5.1 – 10.0	1.8 – 6.5 – 12.4
Stand height (m)	1.2 – 8.6 – 32.6	2.5 – 10.5 – 26.3	1.7 – 12.3 – 31.1
Stocking (trees ha <sup>-1</sup> )	706 – 1143 – 1297	582 – 940 – 1295	398 – 1050 – 1200
Basal area (m <sup>2</sup> ha <sup>-1</sup> )	2.1 – 21.3 – 49.3	0.2 – 19.5 – 33.1	0.0 – 19.9 – 37.1
Volume (m <sup>3</sup> ha <sup>-1</sup> )	2.7 – 76.2 – 336.2	0.2 – 43.3 – 249.3	0.0 – 53.7 – 328.9
Site index (m)	9.7 – 22.7 – 28.8	11.0 – 17.5 – 26.5	7.7 – 16.9 – 27.6

<sup>1</sup>Minimum – mean – maximum

**Table 3.** Summary statistics<sup>1</sup> for the two sets of sample trees used to develop the stem taper model

Statistic	Victoria various	Gippsland
Number of trees	66	189
Age (y)	5 – 8.7 – 30.2	6.4 – 11.0 – 25.1
dbhob (cm)	6.2 – 27.4 – 57.8	7.6 – 18.6 – 34.6
Height (m)	10.0 – 23.4 – 40.0	9.7 – 17.9 – 28.4
Volume (m <sup>3</sup> )	0.011 – 0.676 – 2.959	0.002 – 0.182 – 0.765
Measurements per tree	8 – 13.3 – 19	8 – 16.1 – 26

<sup>1</sup>Minimum – mean – maximum

A basic requirement for growth models based on diameter distributions is the integration of the estimates of total yield of forest stands and the predictions of size class distribution of individual trees (Chang 1987). This is usually achieved by predicting stand attributes using stand-level component models, and then recovering the parameters of a theoretical distribution for tree diameters from the stand attributes predicted at any target age. Various approaches have been developed for such tasks (Vanclay 1994).

Yields may be predicted by using algebraic difference equations (ADE) to project the current stand conditions observed in inventories into the future (Clutter *et al.* 1983). In general, this approach uses the functional form of  $Y_2 = f(Y_1, T_1, T_2)$ , where  $Y_2$  is stand yield predicted at age  $T_2$  years, and  $Y_1$  is the measured yield at age  $T_1$  years. Desirable properties of an ADE model for yield projection include mathematical consistency, projection path invariance and flexibility in functional form (e.g. polymorphic versus anamorphic models). The stand-level component models used in existing growth models for *E. globulus* plantations (i.e. Inions 1992; Tome *et al.* 1997; Wong *et al.* 2000) and for *E. nitens* plantations (e.g. Candy 1997) were constructed using the ADE approach. To estimate an ADE model, the data need to be arranged into measurement pairs. Borders *et al.* (1988) studied the effect of different data arrangements on model estimation. In the present study, all possible measurement pairs with non-decreasing ages were used.

To develop the size class growth model, stand-level component models were first developed to predict the temporal development of stand height, basal area and stocking respectively. Tree-level component models were also developed for estimating the height–diameter relationship, under-bark diameters along the stem (taper) and stem bark thickness of individual trees. For these purposes, various functional (model) forms were initially evaluated, and the best model form was then selected as the base model for each of the modelled stand or tree characteristics. Details of this preliminary modelling and the evaluation of alternative model forms have been reported by Strandgard *et al.* (2005). The base models then selected for each of the stand and tree component models are presented in Tables 4 and 5 respectively.

**Diameter distributions**

The cumulative density function of the three-parameter Weibull distribution was used to model tree diameter distributions over time:

$$p(x) = 1 - \exp \left[ - \left( \frac{x - a}{b} \right)^c \right], \quad (1)$$

where  $x$  represents the response variable (i.e. tree diameter at breast height over bark (dbhob));  $a, b, c$  are the location, scale and shape parameters respectively; and  $a \leq x < \infty$ . The Weibull distribution is superior to other theoretical distributions because

**Table 4.** Functional forms selected for the stand-level component models

Component model	Functional form
Stand height growth/site index	$MDH_2 = \beta_1 \left[ 1 - \left( 1 - \left( \frac{MDH_1}{\beta_1} \right)^{1/\beta_2} \right)^{(T_2/T_1)} \right]^{\beta_2}$
Stand basal area projection	$BA_2 = BA_1^{(T_1/T_2)^{\beta_3}} \times \exp \left[ \left( \frac{\beta_1}{\beta_3} + \frac{\beta_2}{\beta_3} \times S \right) \left( 1 - \left( \frac{T_1}{T_2} \right)^{\beta_3} \right) \right]$
Stand stocking projection	$N_2 = \left[ \frac{1}{\sqrt{N_1}} + \beta_1 \left( \left( \frac{T_2}{100} \right)^2 - \left( \frac{T_1}{100} \right)^2 \right) \right]^{-2}$

**Table 5.** Functional forms selected for the tree-level component models

Component model	Functional form
Height–diameter	$H = 1.3 + \beta_1 \times MDH \left( \frac{1 - \exp(-\beta_2 \times dbhob)}{1 - \exp(-\beta_2 \times MDD)} \right)^{\beta_3}$
Stem taper	$d_h = \beta_0 + \frac{\beta_1}{1 + \beta_2 h} - \beta_3 h - \beta_4 h^2$
Double bark thickness at 1.3 m height	$DBT_{1.3} = \beta_1 \times \exp \left( \beta_2 - \frac{\beta_3}{dbhob} \right)$

of its flexibility in shape and ease of parameterisation (e.g. Bailey and Dell 1973). Weibull distribution-based diameter distributions may be estimated by different methods (Clutter *et al.* 1983). In this study, parameter recovery methods based on moment estimators (ME) (Nanang 1998) and percentile estimators (PE) (Da Silva 1986) were initially evaluated. The PE-based methods were found to be superior and consequently were adopted (see Strandgard *et al.* 2005 for detailed discussions). The method has been used for modelling diameter distributions for various applications in plantations, including testing growth responses to fertiliser application (Bailey *et al.* 1989), studying the impact of inter-specific competition (Knowe 1992) and investigating the effect of site preparation (Knowe and Stein 1995).

The PE-based procedures for modelling diameter distributions over time involve recovering the three parameters of the Weibull distribution analytically from tree diameter percentiles of  $D_0$ ,  $D_{25}$ ,  $D_{50}$ ,  $D_{95}$  and  $D_q$  (see Table 1 for definitions) at the prediction ages:

1. The location parameter is estimated using  $D_0$  and  $D_{50}$ , and the sample size ( $n$ ) (i.e. average number of trees included in the sample plots):

$$\hat{a} = \frac{n^{1/3} D_0 - D_{50}}{n^{1/3} - 1}. \quad (2)$$

2. The shape parameter is estimated using  $D_{25}$  and  $D_{95}$ , and the estimated location parameter ( $\hat{a}$ ):

$$\hat{c} = \frac{2.343088}{\ln(D_{95} - \hat{a}) - \ln(D_{25} - \hat{a})}. \quad (3)$$

3. The scale parameter is estimated using the estimated location and shape parameters and the quadratic mean diameter ( $D_q$ ) by solving the positive root of:

$$\hat{b} = \frac{\hat{a}\Gamma_1}{\Gamma_2} + \sqrt{\left(\frac{\hat{a}}{\Gamma_2}\right)^2 (\Gamma_1^2 - \Gamma_2) + \frac{D_q^2}{\Gamma_2}}, \quad (4)$$

where  $\Gamma_1 = \Gamma(1 + 1/\hat{c})$ ,  $\Gamma_2 = \Gamma(1 + 2/\hat{c})$  and  $\Gamma$  is the Gamma function.

To apply the PE procedure, models are required for predicting  $D_0$ ,  $D_{25}$ ,  $D_{50}$  and  $D_{95}$  at any age of interest. These models are developed by relating the observed percentiles of cumulative distributions of tree diameters to stand characteristics such as MDH, MDD,  $S$ , BA,  $N$  and  $D_q$ .

### Regional differences

The nonlinear extra sum of squares method (Bates and Watts 1988) was used to test for regional differences in the parameters of selected models for stand height, basal area and stocking. The method has been widely used in forestry, for example to test taper equations (Huang 1994), tree volume equations (Pillsbury *et al.* 1995) and tree height–diameter relationships (Huang *et al.* 2000). Application of the method requires fitting the tested models with dummy variables for indexing every region being compared (full model), as well as the tested models with no dummy variable or

dummy variables for indexing two or more regions combined (i.e. restricted or reduced models). The dummy variables are used to separate data in regressions. For example, a dummy variable ( $x$ ) is needed for testing the difference between two regions (i.e.  $x = 1$  if data are from Region 1, otherwise  $x = 0$  for Region 2). The dummy variables are then related to each of the parameters of a tested model using linear equations. The regional difference in the parameters of the tested model is statistically tested using the sum of squared errors of nonlinear regressions for estimating the full and reduced models and associated degrees of freedom respectively.

The appropriate statistic for testing the null hypothesis that there is no difference in the parameters of a tested model between any pairs of regions is:

$$F = \frac{sse(r) - sse(f)}{df(r) - df(f)} \times \frac{df(f)}{sse(f)}, \quad (5)$$

where  $sse(f)$  and  $sse(r)$  are the sum of squared errors from the regressions for the full and reduced models respectively, and  $df(f)$  and  $df(r)$  are the degrees of freedom associated with  $sse(f)$  and  $sse(r)$  respectively. Under the null hypothesis,  $F$  follows an  $F$ -distribution with the degrees of freedom  $df_1 = df(r) - df(f)$  and  $df_2 = df(f)$ . If the resultant  $F \geq F[1 - \alpha; df_1, df_2]$ , where  $F[1 - \alpha; df_1, df_2]$  is the 100(1 -  $\alpha$ ) percentile from an  $F$ -distribution with the degrees of freedom  $df_1$  and  $df_2$ , the null hypothesis is rejected, indicating a significant difference in the parameters of a tested model between the regions. Therefore, the model should be estimated separately for each region. If  $F < F[1 - \alpha; df_1, df_2]$ , the null hypothesis is accepted, indicating no significant difference in the parameters of a tested model between the regions, and the model should be estimated by pooling the data from the regions compared.

For each base component model, the test procedure was used to compare all three regions, and each pair among the three regions. The final stand height, basal area and stocking projection models were determined and parameterised based on the results of the tests.

### Model parameterisation

The underlying statistical assumptions required for justifying regression analysis, including independent and identically distributed (iid) random errors with a normal distribution and constant variances, may not be met when repeated measurements from permanent plots are used (West *et al.* 1984). Thus the parameter estimates of regression models derived using the ordinary least squares (ols) method may be biased and inconsistent, and hypothesis tests performed for determining the best model and predictor variables invalid. To address these problems, the feasible generalised least squares (fgls) method was used whenever initial ols regressions indicated a potential violation of the underlying statistical assumptions. The fgls method allows variance-covariance structures (i.e. heteroscedasticity and/or autocorrelation) to be specified that better model the variability of the data (Greene 1990). The parameter estimates obtained using the fgls method are unbiased, consistent and asymptotically normally distributed. For example, Rivas *et al.*

(2004) used a nonlinear fgls approach to remove the heteroscedasticity of error variances when estimating a stand height growth model for five pine species in Mexico.

The MODEL procedure in the SAS/ETS module (SAS Institute 1993) was used in all nonlinear regression analyses for estimating stand and tree component models. The procedure can perform the nonlinear ols, and also the fgls estimation for a range of variance-covariance structures. The final stand and tree component models were adjudged by the three criteria: (1) all parameter estimates were statistically significant; (2) the residual plot showed no indication of violating the underlying statistical assumptions (i.e. iid errors); and (3) there was generally good agreement between the predicted and observed yield trajectories (values).

## Results

### Regional differences

The base models selected for modelling stand height, basal area and stocking (Table 4) were tested for regional differences. There was no significant difference in the parameters of the stocking base models for the three regions, and consequently further investigation was limited to the stand height and basal area base models, with statistical tests (Tables 6 and 7) concluding that:

- significant differences exist in both stand height and basal area base models across the three regions

- for the stand height base model, significant differences exist between the Green Triangle and Central Victorian regions, and between the Green Triangle and Gippsland regions, but not between the Central Victorian and Gippsland regions
- for the stand basal area base model, significant differences exist between the Gippsland and Green Triangle regions, and between Gippsland and Central Victoria, but not between the Green Triangle and Central Victorian regions.

### Stand height growth and site index model

The statistical tests suggested that separate stand height growth models should be developed for the Green Triangle, and for the Central Victorian and Gippsland regions combined. However, a reliable model could not be derived for the Green Triangle because the available data were limited to stands aged 4 y or younger. Therefore, it was decided to develop a single stand height growth and site index model for use across all three regions. This model can be improved when data become available from older plantations in the Green Triangle. The fitting algorithm was found difficult to converge if the asymptote parameter  $\beta_1$  of the stand height model was not fixed. The problem appears to be associated with this specific model form; Candy (1997) had similar difficulty for the same functional form applied to model stand height for *E. nitens* plantations in Tasmania and New Zealand. To overcome this problem, regressions were performed for a fixed value of parameter  $\beta_1$ , between 40 m and 70 m. The best model was

**Table 6.** *F*-tests for testing the regional differences in the base stand height model (Equation 1, Table 4)

Regions compared <sup>1</sup>	Full model <sup>2</sup>		Reduced model <sup>3</sup>		<i>F</i> -value <sup>6</sup>
	sse <sup>4</sup>	df <sup>5</sup>	sse	df	
GT, CV, GL	7524	2326	8028	2328	78*
GT and CV	1186	1283	1506	1284	346*
GT and GL	6700	1278	7201	1279	96*
CV and GL	7162	2091	7163	2092	0.2

<sup>1</sup>Green Triangle (GT), Central Victoria (CV), Gippsland (GL)

<sup>2</sup>Model fitted with pooled data from the regions being compared

<sup>3</sup>Model fitted with dummy variables for data separation for the regions being compared

<sup>4</sup>sse = sum of squared errors from fitting the reduced or full model

<sup>5</sup>df = degrees of freedom associated with sse

<sup>6</sup>*F*-values estimated using Equation 5, and \* = the regions compared are significantly different ( $P < 0.05$ )

**Table 7.** *F*-tests for testing the regional differences in the base stand basal area model (Equation 2, Table 4)

Regions compared <sup>1</sup>	Full model <sup>2</sup>		Reduced model <sup>3</sup>		<i>F</i> -value <sup>6</sup>
	sse <sup>4</sup>	df <sup>5</sup>	sse	df	
GT, CV, GL	5569	2085	6745	2091	73*
GT and CV	926	1155	931	1158	2.2
GT and GL	4748	1039	5528	1042	57*
CV and GL	5463	1976	5967	1979	61*

<sup>1</sup>Green Triangle (GT), Central Victoria (CV), Gippsland (GL)

<sup>2</sup>Model fitted with pooled data from the regions being compared

<sup>3</sup>Model fitted with dummy variables for data separation for the regions being compared

<sup>4</sup>sse = sum of squared errors from fitting the reduced or full model

<sup>5</sup>df = degrees of freedom associated with sse

<sup>6</sup>*F*-values estimated using Equation 5, and \* = the regions compared are significantly different ( $P < 0.05$ )

adjudged by fit statistics (e.g.  $R^2_{Adj}$  rmse), and accuracy statistics estimated using a cross-validation procedure for prediction (e.g. mean error, mean relative error and mean squared error).

The regression residuals from the ols fit showed no indication of violating the underlying assumptions (Fig. 1a). Therefore, the nonlinear fgls regression was not considered, and the best stand height growth model was:

$$MDH_2 = \hat{\beta}_1 \left[ 1 - \left( 1 - \left( \frac{MDH_1}{\hat{\beta}_1} \right)^{1/\hat{\beta}_2} \right)^{T_2/T_1} \right]^{\hat{\beta}_2}, \quad (6)$$

where  $\hat{\beta}_1 = 50.0$ ,  $\hat{\beta}_2 = 0.8903$ ,  $R^2_{Adj} = 0.88$ , and  $rmse = 1.76$  m (Table 8).

The corresponding site index model was obtained by replacing  $MDH_2$  and  $T_2$  with site index ( $S$ ) and the reference age (i.e. 10 y), respectively:

$$S = \hat{\beta}_1 \left[ 1 - \left( 1 - \left( \frac{MDH}{\hat{\beta}_1} \right)^{1/\hat{\beta}_2} \right)^{10/T} \right]^{\hat{\beta}_2}, \quad (7)$$

where MDH is the stand height measured at any age  $T$  y.

There was generally good agreement between predicted and observed MDH trajectories (Fig. 1b).

**Stand basal area model**

The statistical tests suggested that separate stand basal area models should be developed for Gippsland, and for the Central Victorian and Green Triangle regions combined. An indicator variable ( $x$ ) was used to index the data records and the three parameters in the basal area base model (i.e. Equation 2 in Table 4) were replaced by the following linear functions:

$$\begin{aligned} \beta_1 &= a_0 + a_1x, \\ \beta_2 &= b_0 + b_1x, \\ \beta_3 &= c_0 + c_1x, \end{aligned} \quad (8)$$

where  $x = 1$  if a data record was from Gippsland, and  $x = 0$  if the

data record was from the Central Victorian or Green Triangle region. The extended model was estimated using the MODEL procedure in SAS. Regression residuals from the initial ols fit indicated increasing variance as the initial stand age  $T_1$  increased. A weighting function of  $T_1^{-\alpha}$ ,  $\alpha = 0.5, 1.0, 1.5$  and  $2.0$  was then specified for the nonlinear fgls regressions, and with  $\alpha = 1$  best for improvement of the residual plot (Fig. 1c) and fit statistics.

The parameter estimates for Equation 8 were:

$$\begin{aligned} \hat{\beta}_1 &= 2.4445 - 0.00146x, \\ \hat{\beta}_2 &= 0.05855 - 0.0605x, \\ \hat{\beta}_3 &= 0.9594 - 0.5506x, \end{aligned}$$

with  $R^2_{Adj} = 0.96$  and  $rmse = 1.63$  m<sup>2</sup> ha<sup>-1</sup> (Table 8).

Parameter estimates for the stand basal area model (Equation 2 in Table 4) were thus:

1.  $\hat{\beta}_1 = 2.4431$ ,  $\hat{\beta}_2 = -0.00195$  and  $\hat{\beta}_3 = 0.4088$  for the Gippsland region
2.  $\hat{\beta}_1 = 2.4445$ ,  $\hat{\beta}_2 = 0.05855$  and  $\hat{\beta}_3 = 0.9594$  for the Central Victorian or Green Triangle regions.

In general, there was good agreement between predicted and observed basal growth trajectories, for example as presented in Figure 1d for Gippsland (i.e.  $x = 1$ ) with  $S = 10, 15, 20, 25$  and  $30$  m.

**Stand stocking model**

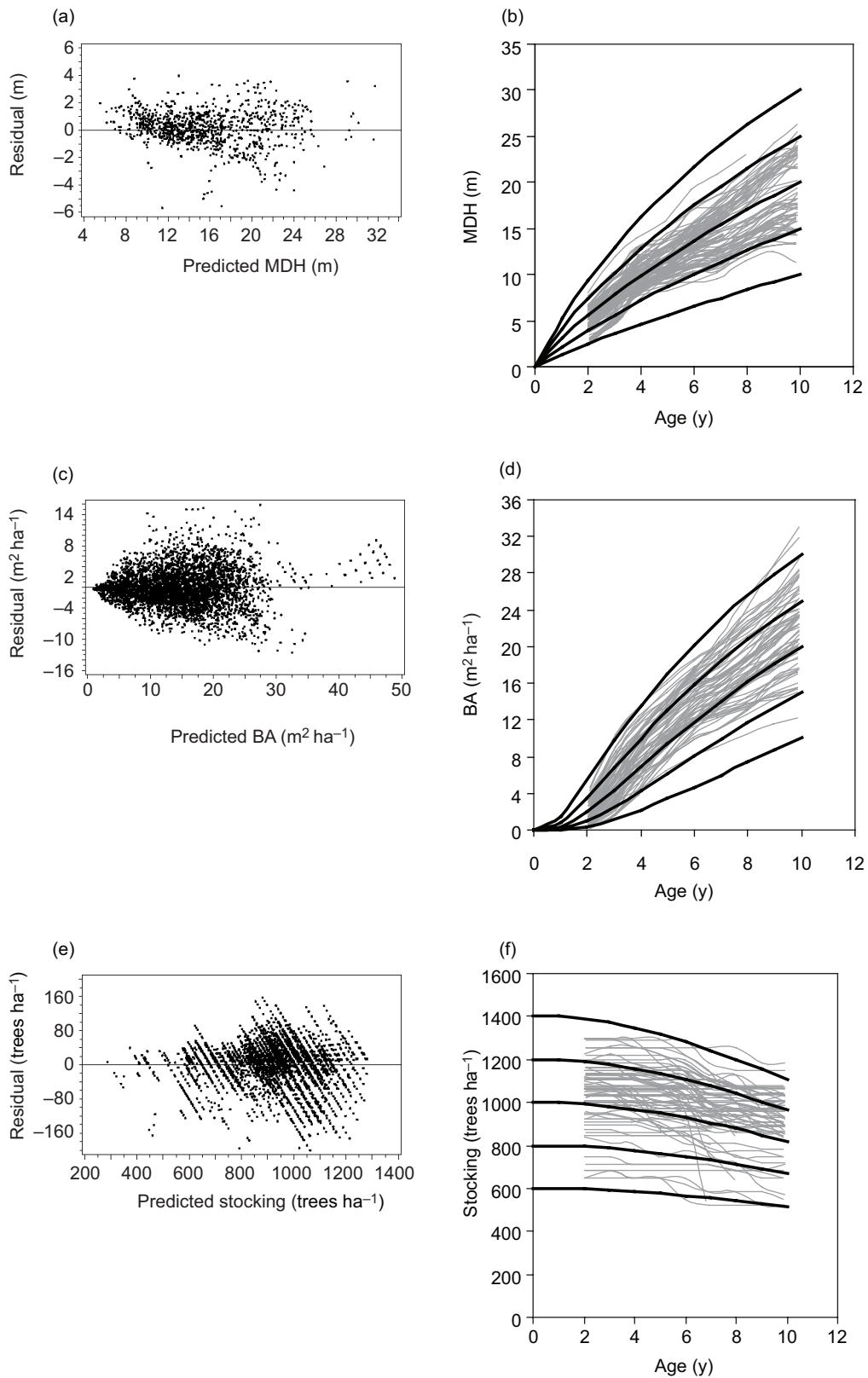
Measurements from the three regions were pooled to estimate the stand stocking projection model (i.e. Equation 3 in Table 4). To test if there was any effect of site quality on stand stocking, a linear function was used to relate the parameter of the model to the site index ( $S$ ) values of individual plots (i.e.  $\beta_1 = a_0 + a_1S$ ). However, the slope parameter estimate was not statistically significant, and the relationship was not subsequently used. The regression residuals derived from the ols fit indicated no violation of the underlying assumptions (Fig. 1e) and therefore the fgls regression was not pursued.

**Table 8.** Parameter estimates and fit statistics for the stand-level component models

Component model	Response variable	Parameter estimate (standard error)				Fit statistic		
		$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$n$	rmse	$R^2_{Adj}$
Stand height (Eqn 6)	MDH <sub>2</sub>	—	50.0 (NA <sup>1</sup> )	0.8903 (0.0101)	—	2170	1.758	0.877
Stand basal area (Eqn 2 in Table 4) <sup>2</sup>	BA <sub>2</sub>	—	2.4445 - 0.00146 $x$ (0.041, 0.003)	0.05855 - 0.0605 $x$ (0.002, 0.006)	0.9594 - 0.5506 $x$ (0.012, 0.060)	2094	1.634	0.962
Stand stocking (Eqn 9)	$N_2$	—	0.3387 (0.009)	—	—	2024	24.111	0.977
Diameter percentiles (Eqn 17)	$D_0$	3.8386 (0.264)	0.02863 (0.002)	-0.1001 (0.030)	-0.06909 (0.016)	1245	2.012	0.555
	$D_{25}$	7.6138 (0.203)	0.03867 (0.002)	-0.1164 (0.011)	-0.06122 (0.006)	1245	1.452	0.855
	$D_{50}$	8.7702 (0.162)	0.05166 (0.001)	-0.1423 (0.009)	-0.06640 (0.005)	1245	1.239	0.921
	$D_{95}$	10.9861 (0.203)	0.06992 (0.002)	-0.1944 (0.011)	-0.06579 (0.006)	1245	1.591	0.918

<sup>1</sup>Standard error not available when a parameter was fixed during fitting

<sup>2</sup> $x$  is the region indicator: Gippsland ( $x = 1$ ), Central Victoria ( $x = 0$ ) and Green Triangle ( $x = 0$ )



**Figure 1.** Stand height (MDH), basal area (BA) and stocking ( $N$ ) component models: (a), (c) and (e) regression residuals versus predicted values; and (b), (d) and (f) observed versus predicted growth trajectories for different initial values

The estimated stand stocking projection (mortality) model was:

$$N_2 = \left[ \frac{1}{\sqrt{N_1}} + \hat{\beta}_1 \left( \left( \frac{T_2}{100} \right)^2 - \left( \frac{T_1}{100} \right)^2 \right) \right]^{-2}, \quad (9)$$

where  $\hat{\beta}_1 = 0.3387$ ,  $R^2_{Adj} = 0.98$ , and  $rmse = 24.1$  trees  $ha^{-1}$  (Table 8).

Stand stocking predictions for the initial stocking of 600, 800, 1000, 1200 and 1400 trees  $ha^{-1}$ , overlain with the observed stocking trajectories, are presented in Figure 1f.

### Tree height–diameter model

A relationship between tree height and diameter is required where actual or predicted diameter distributions are the basis for calculation of individual tree volumes (and then by summation to total stand volume, or volume by size classes) using either a tree volume or stem taper equation. Stand characteristics such as age, stocking, basal area or dominant height can improve tree height–diameter relationships (e.g. Bi *et al.* 2000; Staudhammer and LeMay 2000).

Alternative functional forms were initially evaluated for estimating the tree height–diameter relationship (see Strandgard *et al.* 2005 for detailed discussion). For each of the model forms evaluated, different strategies were tested to include stand variables to improve predictions of tree height. In general, stand height (MDH) or site index ( $S$ ), and dominant diameter (MDD, Table 1) improved predictions of tree height, but stand age, stocking or basal area did not. The following modification of the Chapman-Richards function after Bi *et al.* (2000), i.e. Equation 1 in Table 5, was found to be best:

$$H = 1.3 + \hat{\beta}_1 \times MDH \left( \frac{1 - \exp(-\hat{\beta}_2 \times dbhob)}{1 - \exp(-\hat{\beta}_2 \times MDD)} \right)^{\hat{\beta}_3}, \quad (10)$$

where  $\hat{\beta}_1 = 0.9148$ ,  $\hat{\beta}_2 = 0.1504$ ,  $\hat{\beta}_3 = 1.6079$ ,  $R^2_{Adj} = 0.97$  and  $rmse = 1.01$  m (Table 9). The regression residuals showed no indication of violating the underlying assumptions (Fig. 2a). In this model, the intercept is fixed at 1.3 to account for tree  $dbhob = 0$  cm at  $H = 1.3$  m.

In the developed height–diameter relationship, the predictor variables of MDH and MDD provide measures of site productivity and stocking respectively; the prediction of tree height can thus be improved over different site and stocking

conditions. For a given dbhob, the predicted height increases as MDH increases, but decreases as MDD increases. It must be noted that the height–diameter relationship was developed using tree data sampled mainly from plantations with diameters and total heights of trees smaller than 35 cm and 30 m respectively, and height predictions for trees outside these ranges need to be validated.

### Tree stem taper model

Alternative functional forms of stem taper models were initially examined, including those of Max and Burkhart (1986) and Kozak (1988). The extended segmented taper model suggested by Goodwin and Thompson (2003) was found to be the most appropriate for the available data, and was selected as the base model for further evaluation (i.e. Equation 2 in Table 5).

To determine the parameters of the base stem taper model, Goodwin and Thompson (2003) first related the parameters  $\beta_1$  and  $\beta_3$  analytically to tree height ( $H$ ). This was realised by imposing the constraints that when  $h = H$ ,  $d_h = 0$  (i.e. the predicted stem diameter is zero if the sectional height input for prediction is total height), and that when  $h = h_1$ ,  $d_h = d_1$  (e.g.  $h = 1.3$  m, then  $d_h = dbhob$ ), where  $h_1 \neq H$ . These constraints gave:

$$\beta_0 = -\frac{\beta_1}{1 + \beta_2 H} + \beta_3 H + \beta_4 H^2, \quad (11)$$

$$\beta_3 = \frac{dbhob}{H - 1.3} - \frac{\beta_1 \beta_2}{(1 + 1.3\beta_2)(1 + \beta_2 H)} - \beta_4 (H + 1.3). \quad (12)$$

They then related the parameters  $\beta_1$ ,  $\beta_2$  and  $\beta_4$  to tree height through the best species-specific relationship selected using a two-stage modelling approach. For *E. globulus* plantations in Tasmania, the relationships selected were:

$$\beta_1 = \gamma_1 H, \quad (13)$$

$$\beta_2 = \frac{\gamma_2}{H}, \quad (14)$$

$$\beta_4 = \lambda_0 + \frac{\lambda_1}{H}, \quad (15)$$

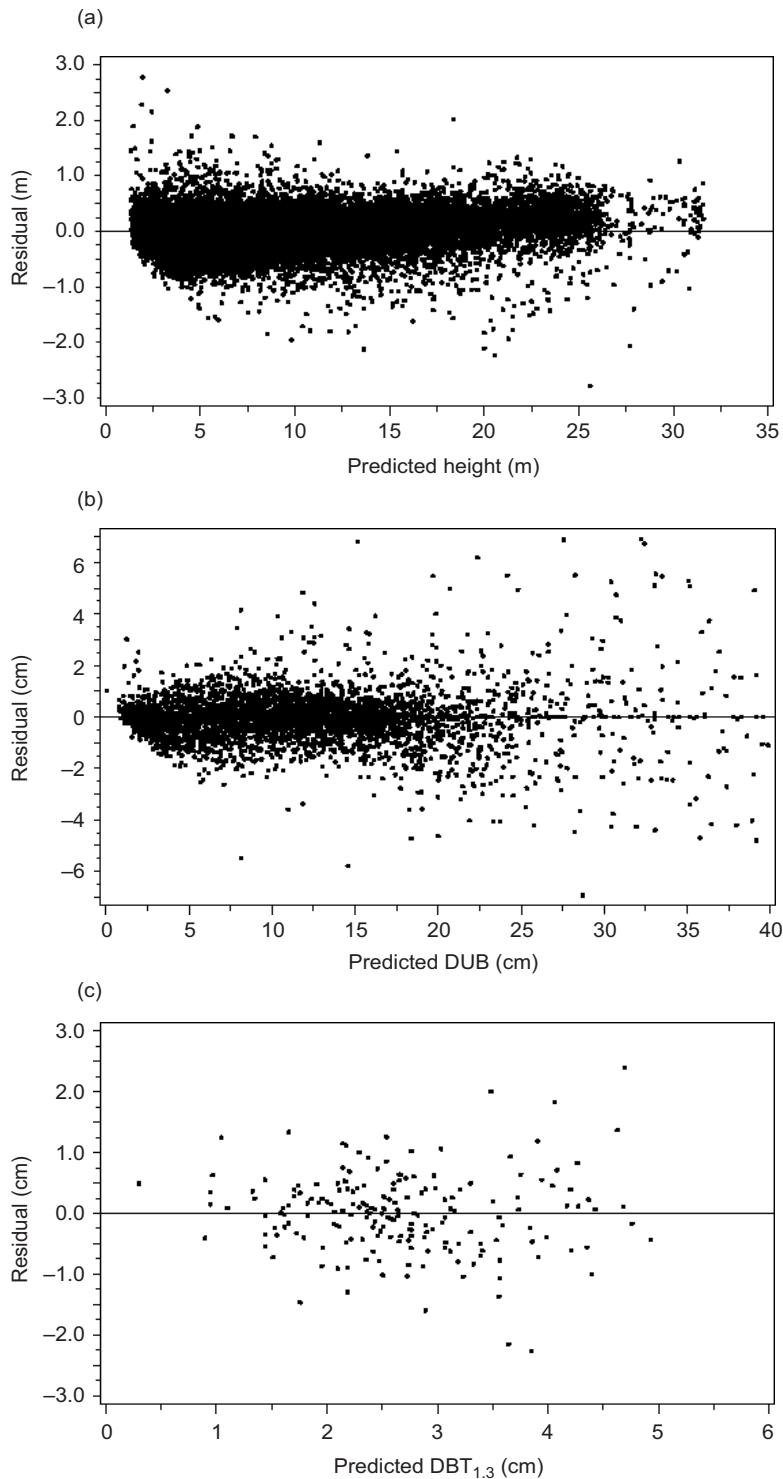
where  $\gamma_1$ ,  $\gamma_2$ ,  $\lambda_0$ , and  $\lambda_1$  are parameters to be estimated.

**Table 9.** Parameter estimates and fit statistics for tree-level component models (see Table 5 for model forms)

Component model	Response variable	Parameter estimate (standard error)					Fit statistic		
		$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$n$	rmse	$R^2_{Adj}$
Height–diameter	H	—	0.9148 (0.001)	0.1504 (0.002)	1.6079 (0.012)	—	25831	1.007	0.971
Stem taper	$d_h$	Eqn 11 <sup>1</sup>	0.4459 H (0.005)	85.3142 / H (4.231)	Eqn 12 <sup>2</sup>	0.0228 + 0.4185 / H (0.023, 0.042)	4001	1.118	0.983
Double bark thickness	DBT <sub>1.3</sub>	—	1.0511 (0.051)	1.7910 (1.135)	19.5889 (3.241)	—	191	0.659	0.618

<sup>1</sup>The taper model parameter  $\hat{\beta}_0$  is estimated using Equation 11

<sup>2</sup>The taper model parameter  $\hat{\beta}_4$  is estimated using Equation 12



**Figure 2.** Regression residuals graphed against the predicted values for (a) tree height-diameter model (Equation 1, Table 5), (b) stem taper model (Equation 2, Table 5), and (c) bark thickness model (Equation 3, Table 5)

In the present study, alternative relationships for the parameters  $\beta_1$ ,  $\beta_2$  and  $\beta_4$  were firstly compared with those selected by Goodwin and Thompson (2003, i.e. Equations 13–15) using the available data. This was implemented by estimating the base model (Equation 2 in Table 5) and the relationships specified for  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$  and  $\beta_4$  (i.e. Equations 11–15) using the nonlinear simultaneous equations fitting procedure in the SAS/ETS

application (SAS Institute 1993). The constraint,  $h_1 = 1.3$  m, and  $d_1 = \text{dbh}$  were used in all fittings. The results confirmed that Equations 13–15 were also appropriate and the parameter estimates for these relationships were:

$$\begin{aligned}\hat{\beta}_1 &= 0.4459H, \\ \hat{\beta}_2 &= \frac{85.3142}{H}, \\ \hat{\beta}_4 &= 0.02280 + \frac{0.4185}{H}.\end{aligned}$$

The resultant stem taper model had  $\text{rmse} = 1.12$  cm, and  $R^2_{\text{Adj}} = 0.98$  (Table 9). The residual plot is shown in Figure 2b.

In application, the parameters  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ , and  $\beta_4$  will be first predicted from the observed  $H$  and  $\text{dbh}$  of a sample tree using Equations 11–15. The predicted parameters are then applied to the taper model (Equation 2 in Table 5) for estimating the under-bark stem diameters along the stem, and corresponding sectional volumes. When the sectional height to be predicted is total height, the predicted volume will be total under-bark volume of a sample tree. To validate the model, we showed in a separate study using a sequential accuracy testing approach (Wang and Baker 2005) that the predictions of under-bark stem diameters derived from this stem taper model would have an acceptable accuracy for most expected applications.

### Tree stem bark thickness model

Tree  $\text{dbh}$  and total height are required as inputs to the developed stem taper model for predicting underbark stem diameters.  $\text{Dbh}$  is estimated as the difference between  $\text{dbh}$  and double bark thickness ( $\text{DBT}_{1,3}$ ), but because the latter is not commonly measured during inventory, a bark thickness prediction model is then required. The best model found for this purpose based on the available data was:

$$\text{DBT}_{1,3} = \hat{\beta}_1 \times \exp\left(\hat{\beta}_2 - \frac{\hat{\beta}_3}{\text{dbh}}\right), \quad (16)$$

where  $\hat{\beta}_1 = 1.0511$ ,  $\hat{\beta}_2 = 1.7910$ ,  $\hat{\beta}_3 = 19.5889$ ,  $R^2_{\text{Adj}} = 0.62$ , and  $\text{rmse} = 0.66$  cm (Table 9). The residual plot is shown in Figure 2c.

### Diameter percentiles models

Quadratic mean diameter ( $D_q$ ), stand basal area (BA) and age ( $T$ ) were identified as the best stand variables for predicting the percentiles  $D_0$ ,  $D_{25}$ ,  $D_{50}$  and  $D_{95}$  of the cumulative diameter distribution at any age. The base model selected on the basis of fit statistics and residual analysis was:

$$\hat{D}_{\text{pct}} = [\hat{\beta}_0 + \hat{\beta}_1 D_q \times T][1 - \exp((\hat{\beta}_2 + \hat{\beta}_3 \text{BA})T)], \quad (17)$$

where  $\hat{D}_{\text{pct}}$ , is the predicted value for  $D_0$ ,  $D_{25}$ ,  $D_{50}$  or  $D_{95}$ .

The fit statistics and parameter estimates of the prediction models are presented in Table 8. In application, BA and  $N$  at age  $T$  are first predicted from the stand basal area and stocking projection models respectively (i.e. Models 2 and 3 respectively in Table 4).  $D_q$  is then calculated based on the predicted BA and  $N$ .

**Model application**

The developed stand- and tree-level component models, together with the models for predicting the four percentiles of diameter distributions can be used to construct a whole-stand distribution growth and yield model (Vanclay 1994, p. 23). This size-class growth model can be used for projecting the characteristics of plantations sampled in inventories into the future for wood flow analysis and management planning. The procedures are:

1. Estimate the site index ( $S$ ) for a sample plot or site from the stand height (MDH) measured at age  $T$  y using the site index model (Equation 7).
  2. Set the measured stand height, basal area and stocking, the estimate of site index and the inventory age to be the initial stand conditions (i.e. MDH<sub>1</sub>, BA<sub>1</sub>,  $N_1$  and  $T_1$ ) respectively for the three stand-level models (i.e. Equations 1–3 in Table 4).
  3. Predict stand height, basal area and stocking at the prediction age  $T_2$  (i.e. MDH<sub>2</sub>, BA<sub>2</sub>, and  $N_2$ ) from the stand-level models respectively with their parameter estimates (Table 8).
  4. Estimate stand volume ( $V_2$ ) at  $T_2$  using the following stand volume equation (Wong *et al.* 2000):
- $$V_2 = 0.3983 \times BA_2 - 0.0661 \times MDH_2 + 0.35366 \times BA_2 \times MDH_2. \tag{18}$$
5. Estimate the diameter percentiles at age  $T_2$  ( $D_0, D_{25}, D_{50}$  and  $D_{95}$ ) using Equation 17 and their parameter estimates respectively (Table 8), where  $BA = BA_2$ ,  $T = T_2$ , and  $D_q = \sqrt{(40\,000/\pi) (BA_2/N_2)}$ .
  6. Predict the location and shape parameters of the Weibull distribution at age  $T_2$  ( $a$  and  $c$ ) using Equations 2 and 3. Negative values for  $a$  should be set to zero.
  7. Solve Equation 4 for the Weibull scale parameter  $b$  (i.e. positive root of Equation 4) using the estimated  $a$ ,  $c$  and  $D_q$ .
  8. Predict the probabilities ( $f_i$ ) of diameter occurrence by 1 cm intervals of tree dbhob as:

$$f_i = \exp \left[ - \left( \frac{L_i - a}{b} \right)^c \right] - \exp \left[ - \left( \frac{U_i - a}{b} \right)^c \right], \tag{19}$$

where  $L_i$  and  $U_i$  are the lower and upper limits respectively

for  $i$ th diameter class  $i = 1, \dots, k$ , and  $k$  is the total number of diameter classes. In application, the lower limit of the first diameter class should be set to the location parameter  $a$  (i.e.  $L_1 = a$  and  $U_1 = a + 1$ ), which corresponds to the estimated minimum tree diameter, and  $k$  should be determined with the condition of  $\sum_{i=1}^k f_i = 1$ .

9. Estimate the stocking for each diameter class by multiplying the estimated  $f_i$  with the stand stocking,  $N_2$ , predicted as at Step 3.
10. Estimate the volumes for each diameter class. For this, mean volume for a tree corresponding to the mid-point of each diameter class can be determined using the developed height–diameter relationship (Equation 10) and taper model (Equation 2 in Table 5 with parameter estimates as in Table 9).
11. Reset the initial conditions (predictor variables) at age  $T_1$  of the stand-level models to be the predicted values at age  $T_2$  obtained in Step 3 (i.e. MDH<sub>2</sub>, BA<sub>2</sub>,  $N_2$ ), and prediction age to be  $T_2 + 1$  (if annual predictions are required), and repeat Steps 2 to 10 until the target age or prediction interval is reached.

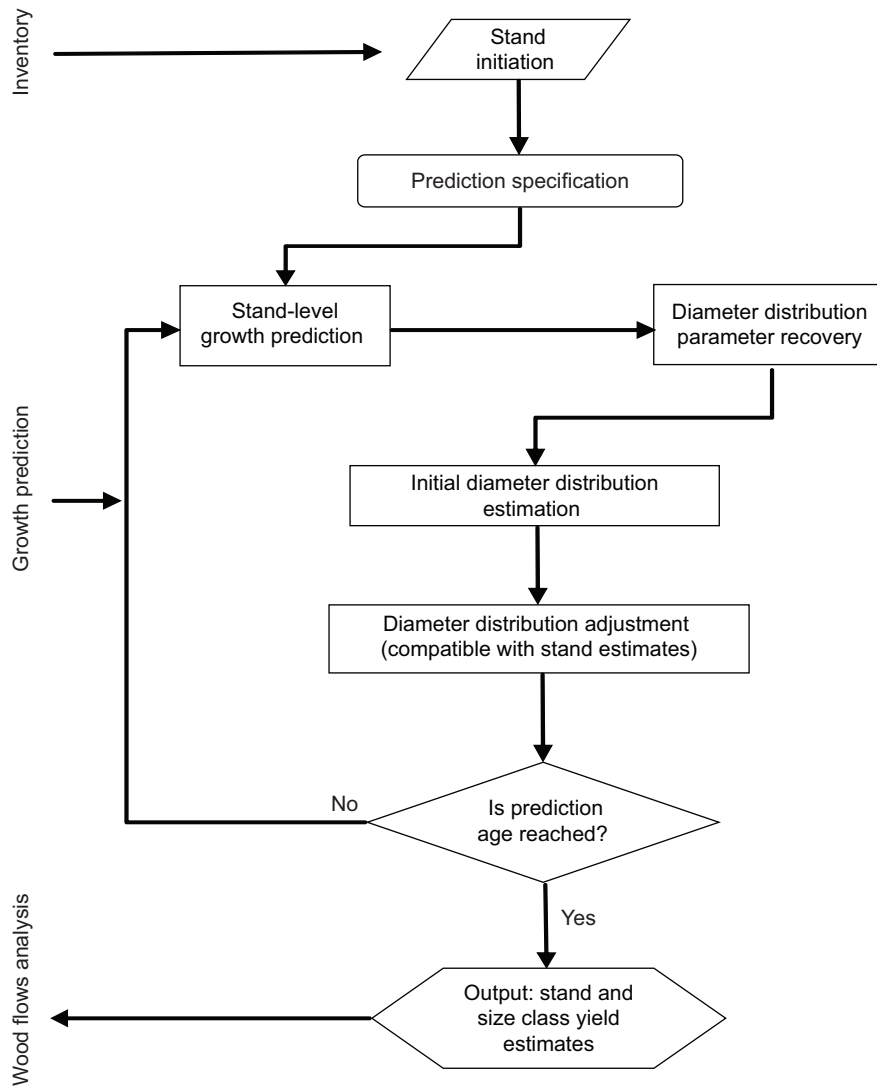
In this procedure, the predictions from the stand-level models are used as a control, and the stand stocking, basal area and volume derived from the predicted diameter distributions are adjusted to be compatible with the stand-level predictions (Fig. 3). This approach has been widely used (e.g. Nepal and Somers 1992; Cao and Baldwin 1999; Wang and Hamilton 2003), and an appropriate algorithm and computer program is required for iterative adjustments.

**Accuracy evaluation**

The prediction accuracy of the stand growth models developed in the present study was evaluated using measurements from the reserved permanent plot data set described earlier. Stand (dominant) height, dominant diameter (MDD) and basal area values were calculated for every combination of plots and measurements. Volumes of individual trees were estimated using the height–diameter relationship developed in this study, and an existing tree volume equation (Wong *et al.* 1999). Stand volumes for each combination of plots and measurements were the sum of volumes of individual trees. Because stand-level models developed have the algebraic difference equation form, the calculated stand characteristics were rearranged into measurement pairs with non-decreasing ages for yield predictions. Summary statistics for the last measurement data from the 338 permanent plots used are presented in Table 10.

**Table 10.** Summary statistics for the last measurements of permanent sample plots ( $n = 338$ ) used for accuracy evaluation

Statistic	Age (y)	MDH (m)	BA (m <sup>2</sup> ha <sup>-1</sup> )	V (m <sup>3</sup> ha <sup>-1</sup> )	MAI (m <sup>3</sup> ha <sup>-1</sup> )	S (m)
Mean	5.8	11.3	13.4	61.3	10.5	16.9
Minimum	4.9	4.6	2.2	6.3	1.2	8.0
Maximum	7.0	18.6	29.4	170.1	34.2	25.6



**Figure 3.** Flowchart illustrating application of the developed growth model

**Table 11.** Overall accuracy of predictions derived from stand growth models ( $n = 1997$ )

Accuracy measure <sup>1</sup>	Mean	Minimum	Maximum
Stand height:			
MD (m)	-0.66	-5.85	4.13
MPD (%)	-7.34	-68.72	28.32
MSD	2.82	0.05	33.61
Stand basal area:			
MD (m <sup>2</sup> ha <sup>-1</sup> )	0.31	-3.96	7.48
MPD (%)	1.56	-63.53	45.91
MSD	2.28	0.05	33.96
Stand volume:			
MD (m <sup>3</sup> ha <sup>-1</sup> )	1.01	-36.4	46.1
MPD (%)	1.06	-86.52	56.21
MSD	115.91	0.04	607.53

<sup>1</sup>MD = Mean difference between observed and predicted values  
 MPD = Mean percent difference between observed and predicted values  
 MSD = Mean squared difference between observed and predicted values

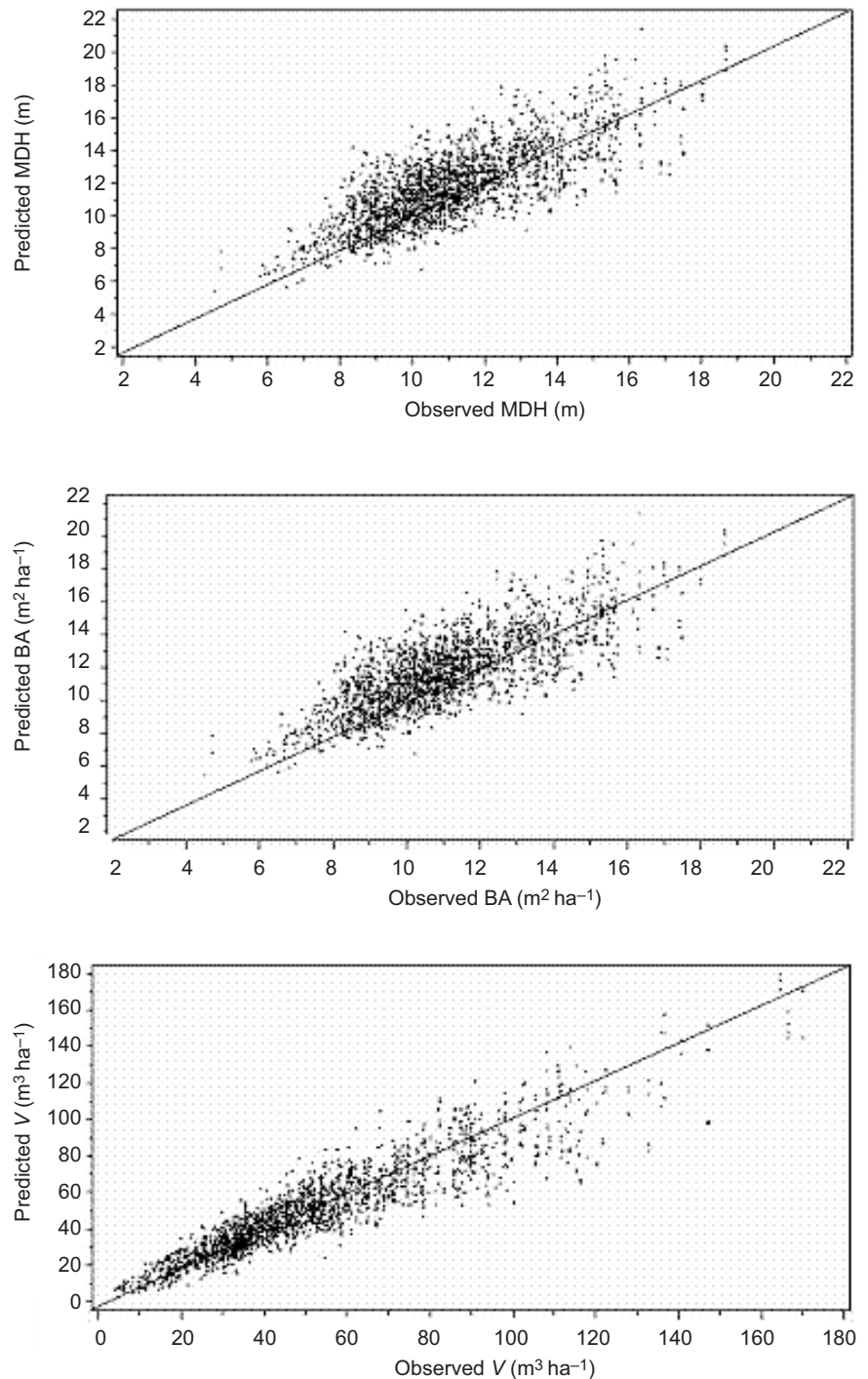
For each stand characteristic evaluated, three statistics were used to measure the prediction accuracy (Table 11):

1. Mean difference (bias) ( $MD = \Sigma(y_i - \hat{y}_i) / n$ ), where  $y_i$  and  $\hat{y}_i$  are the measured and predicted values respectively, and  $n$  is the total number of measurement pairs used for the comparison. MD measures the overall unbiasedness of predictions.
2. Mean percent difference (MPD(%) =  $[\Sigma(y_i - \hat{y}_i) / y_i] \times 100 / n$ ). MPD measures the overall mean bias in a relative unit.
3. Mean squared difference (MSD =  $\Sigma(y_i - \hat{y}_i)^2 / n$ ). MSD measures the total accuracy of prediction, which accounts for both unbiasedness and precision of prediction (Reynolds 1984).

Mean bias for predicted stand heights was relatively small ( $-0.7$  m), mean per cent bias was about  $-7.3\%$  of the observed values, and the overall accuracy, accounting for both bias and imprecision was 2.8. Similarly, mean bias in predicted stand basal areas was relatively small ( $0.3$   $m^2$   $ha^{-1}$ ), and about  $1.6\%$  of the observed values with an overall accuracy of 2.3. Mean bias in predicted stand volumes was  $1.0$   $m^3$   $ha^{-1}$ , about  $1.1\%$  of observed values. The corresponding MSD was 116; that indicates relatively high variation in predicted stand volume. However, for a prediction interval of 5 y or less, average bias in predictions should be expected to be within  $\pm 1.0$  m for stand height,  $\pm 0.5$   $m^2$   $ha^{-1}$  for basal area, and  $\pm 1.0$   $m^3$   $ha^{-1}$  for stand volumes, if the developed models are applied to predict future wood yields for a plantation population based on the inputs (current stand characteristics) observed in a large number of sample plots established across the population. Graphs of predicted versus observed stand height, basal area and volume for the stand-level component models applied to the validation data set are presented in Figure 4.

## Discussion

The stand- and tree-level models developed here for *E. globulus* plantations are the best representation of the long-term growth measurements (yield trajectories) from the Gippsland, Central Victorian and Green Triangle regions that were made available to the present study. Predictions derived from the developed stand-level models represent the mean growth for plantations: (i) in the range of climatic (e.g. rainfall distribution patterns) and edaphic conditions (dominantly former agricultural sites) sampled in these regions, (ii) established and managed with similar silvicultural practices (planting stocking 1000–1200 trees  $ha^{-1}$ , unthinned), and (iii) for the age range represented by the data (less than about age



**Figure 4.** Predicted versus observed stand height (MDH), basal area (BA) and volume (V) for the stand-level component models applied to the validation data set

12 y), used in model selection and parameterisation. These models should be validated before being applied to plantations grown under different conditions. Wang and LeMay (1996) and Wang *et al.* (1996) developed statistical testing approaches for such applications.

The stand yields predicted from the developed models generally agreed well with observed values from the permanent sample plots, which are independent of model parameterisation. Accuracy evaluation indicated that for a prediction interval of 5 y or less,

the average biases in predictions were generally small and within 8.0% of measured stand heights, 3.0% of measured basal areas and 2.0% of measured stand volumes. Thus these models are reliable for projecting mid-rotation (i.e. 5–6 y) inventory data to rotation age (i.e. about 10 y) for wood flow analysis and strategic or management planning. Greater accuracy is expected for predictions over intervals 2–3 y from pre-harvest inventory for tactical planning of wood flows.

The stand and tree models, together with the Weibull distribution-based diameter distribution model, are useful for deriving yield estimates by size (dbhob) classes. This information is essential for analysing the structure of managed plantations and making management decisions, particularly those associated with product assortment, recovery and harvesting costs.

The models may be extended to simulate the response of *E. globulus* plantations to a range of silvicultural options for determining optimal management regimes. However, additional component models need to be developed for stand initiation under a range of silvicultural treatments (e.g. different planting densities). Also, models that can predict growth responses to different silvicultural treatments, particularly spacing and thinning, need to be developed using growth data from permanent plots in experimental trials.

The models can also be extended to incorporate the key environmental variables that are known to affect tree growth, so providing improved site-specific predictions and providing sensitivity to variation in climate. For example, Wang *et al.* (2007) used a non-linear mixed-effects modelling approach to incorporate rainfall and temperature into a stand height growth model for *E. globulus* plantations in Central Victoria.

The development of stand growth models should be undertaken as a continuous process in order to make use of new data from permanent sample plots representing different site and silvicultural conditions, and reflecting the results of tree improvement. We suggest that where there is a diversity of plantation investors, owners and managers such as there is for *E. globulus* in southern Australia, model development and validation can be undertaken most efficiently as a collaborative effort and that publication contributes generally to long-term confidence in this industry.

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