Relative efficiency of projecting basal area and height versus projecting volume in a plantation planning system

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Summary

Growth models are developed for unthinned stands of radiata pine in the south-east of South Australia using data from the Permanent Sample Plots maintained over many years by the Woods and Forests Department, now ForestrySA. The data base is one of the few that can be used to investigate whether it is better to project basal area and height separately or project volume directly in a plantation planning system.

It is concluded that it is marginally more accurate to predict volume growth than it is to predict basal area growth and upper stand height growth before converting to volume. The latter approach is most commonly used in forest management planning in Australasia and elsewhere.

Keywords: forest management; planning; growth models; prediction; basal area; height; volume; *Pinus radiata*

Introduction

During forest inventory it is common to measure only tree diameters (or basal area, *B*) and upper stand height (*H*), but estimates of volume are needed for forest planning. One common approach is to use tree taper equations as in the MARVL style of inventory pioneered in New Zealand (Goulding and Murray 1976; Goulding and Lawrence 1992; Gordon and Lawrence 1995). This can provide product information at inventory. An alternative is to use tree or stand volume equations. Planning systems then predict future growth with basal area increment generally being predicted and apportioned to the individual trees. Height increment may also be apportioned. Many variants on this general model are possible, but most forest owners follow this approach of growing basal area and height, and then estimating volume in their planning systems.

In South Australia the approach adopted by N.W. Jolly and later N.B. Lewis (Lewis *et al.* 1976; O'Hehir *et al.* 2000) in planning systems has been to grow volume instead of basal area and upper stand height. Lewis (*pers. comm.*) suggests that this was because height growth was very variable and therefore difficult to model for the wind-swept coastal plains in the south-east of South Australia, where there are also small micro-topographic effects. Leech (1978) developed the volume-based stand growth models that are currently used to predict future outturn from the radiata pine (*Pinus radiata* D.Don) plantations in South Australia. During that analysis a number of growth models for basal area and upper stand height were also developed, but these were not reported.

There are therefore two alternative general modelling lines possible. The question can be posed as to which is better.

Alternative 1: At inventory, measure basal area and upper stand height, then grow basal area and upper stand height into the future, apportion increment to trees, calculate tree volumes based on taper equations or tree volume equations, and summarise.

Alternative 2: At inventory, measure basal area and upper stand height, calculate tree volumes at time of inventory based on taper equations or tree volume equations, then grow stand volume into the future and apportion volume increment to trees, and summarise.

The basic difference between these two general approaches is whether basal area and upper stand height growth, or volume growth, are predicted in the modelling process. There are many relatively minor differences in the way each alternative might be applied in practice.

The decision as to which alternative to adopt in developing planning systems would generally seem to have been based simply on the availability of data to develop the necessary underpinning models. Private discussions with staff of a number of organisations suggest that the method in use was selected simply because that was the only methodology that was feasible at the time in their organisation, and that the alternatives were not considered.

The apparently simple question as to which alternative is better has rarely been addressed. South Australia is one of the few places in the world where there are data available that enable this question to be examined.

Modelling strategy

To answer the question, the critical element is the necessity to have consistent and independent measurements of basal area, upper stand height and volume, measured over many years to consistent mensuration standards. It is necessary for the volume measurements to be based not on simple tree or stand volume equations or generalised taper equations, but on independent measurements. The Permanent Sample Plot data base from the Woods and Forests Department in South Australia, now ForestrySA, is one of the few data bases that comes close to meeting these requirements.

Given an appropriate data base, a number of models then need to be developed using the available data. These include models to predict stand volume from stand basal area and upper stand height, and models to predict future growth of volume, basal area and upper stand height.

Stand growth models

Many different structural forms are possible for a stand growth model, but studies such as those by Leech (1978) and Leech and Ferguson (1981) indicate that there is often not a great deal of difference in the ability of apparently contradictory models to predict future growth. For radiata pine plantations in the south-east of South Australia, the von Bertalanffy family of models was accepted as the best of the many forms tested. A history of the development of growth models in South Australia is provided by O'Hehir *et al.* (2000).

Von Bertalanffy (1942, 1957, 1969) developed a general equation based on the theory that growth is represented by anabolic growth rate (constructive metabolism) and catabolic rate (destructive metabolism), both of which are allometric in form. If Y is yield at age A then this results in a four-parameter differential equation:

$$\frac{\mathrm{d}Y}{\mathrm{d}A} = nYm - pYr\,,$$

which cannot be integrated, although if it is assumed that ($r = 1$) then it can be integrated to give:

$$Y = \left(\frac{n}{p}\right)^{1/(1-m)} \left[1 - \exp\left(-\,p(1-m)(A - a0)\right)\right]^{1/(1-m)},$$

where $a0$ is the constant of integration, and can be interpreted as the age at which growth commences.

Various forms of this have been developed including the so-called Chapman–Richards form (Richards 1959; Chapman 1961; Pienaar and Turnbull 1973).

$$Y = c_1\left[1 - \exp\left(-\,c_2(A - c_3)\right)\right]^{c_4}.$$

Both forms reach an asymptotic maximum $Y_{\max}$

$$Y_{\max} = \left(\frac{n}{p}\right)^{1/(1-m)} = c_1$$

and both forms have four parameters to be estimated. The Chapman–Richards form appears simpler but correlation between the parameters generally makes it more difficult to efficiently determine the parameters. Leech (1976, 1978) showed that for radiata pine the parameters are less correlated if the von Bertalanffy form is used.

Leech (1978) also found that the constrained form to estimate Y_A, yield at age A,

$$Y_A = Y_{10}\left[\frac{1 - \exp(-p(1-m)(A - a0))}{1 - \exp(-p(1-m)(10 - a0))}\right]^{1/(1-m)}$$

was the best form for predicting volume growth, where site potential (Y_{10}) was defined as the value of yield at a base age of 10 y. This has the advantage of reducing the number of parameters to be estimated to three, p, m and $a0$. For this study V_{10}, B_{10} and H_{10} represent the appropriate age 10 variables for volume, basal area and predominant height respectively. The variables V_A, B_A and H_A represent the values of the same variables at age A.

The modelling strategy used by Leech (1978, Ferguson and Leech 1978) fitted models to each plot with sufficient observations, and then in a second-stage analysis the parameters were related to the volume-based index of site potential. Leech found that the best approach was to use this constrained form as a difference equation, estimating periodic increment. He also used generalised least squares (GLS) techniques (Ferguson and Leech 1978) to overcome the statistical difficulties of the use of repeated measurements in the analysis. Later discussions (including Davis and West 1981; West *et al.* 1986) suggest that if the model structure is fixed and if hypothesis tests are not being carried out to test the significance of each parameter, then other simpler methods are just as appropriate.

For this study it was decided to fit the yield curves to each data set using the constrained yield model, and then to evaluate site potential indices, defined as total production volume at age 10 y, total production basal area at age 10 y and predominant height at age 10 y, as the appropriate constraining variables. This parallels the current use of site potential in South Australia (Lewis *et al.* 1976).

Given the range of the available data — the first measurement of volume were made after age 9 y — it was unlikely that all three parameters would be able to be estimated in the first stage. For volume, it was likely that the simpler Mitscherlich model (Mitscherlich 1910) with ($m = 0$) would be best, as this was found to be so by Leech (1978) using both thinned and unthinned plots. For basal area, it might be possible to estimate all three parameters (p, m, $a0$), but it was likely one would become redundant. For predominant height, setting ($a0 = 0$) is logical and the data suggest that ($m = 0$) was also a possibility, although all three parameters could be evaluated.

Volume prediction model

The combined variable tree volume equation model predicts tree volume from the product of the square of diameter and tree height (D^2H) and the equivalent stand-based model predicts stand volume (V) from the product of basal area (B) and upper stand height or predominant height (H):

$$V = b_0 + b_1\,BH\,.$$

Experience in South Australia suggests that the powers should also be allowed to fluctuate:

$$V = b_0 + b_1 \, B^{b_2} H^{b_3} \, .$$

The parameter b_2 is generally within the range 2.2–2.5 and b_3 generally 0.5–0.8, which is consistent with dimension analysis (Khil'mi 1957) of an approximately second-degree paraboloid. Further, the tree volume equation is generally best developed as a weighted model with the weight approximately proportional to the volume itself:

$$Vw = b_0 \, w + b_1 \, B^{b_2} H^{b_3} \, w \, ,$$

$$w = \frac{1}{BH} \, .$$

Other models could also be evaluated.

Data

The South Australian Permanent Sample Plot (PSP) data base has a long history of reliable, consistent measurements, including sample tree measurements of volume so that the volume measurements can be considered independent of basal area and upper stand height measurements. The plot series was commenced by N.W. Jolly in 1935, was later the responsibility of N.B. Lewis from 1950, then A. Keeves until the late 1970s, and latterly I.B. Millard, M.W. Sutton, J.F. O'Hehir and J. Rombouts. The plots have been measured to generally consistent standards throughout by well-trained and experienced technical staff.

Plots were generally 0.08–0.1 ha. Plots were either left unthinned or thinned to various thinning regimes. The 12 plots selected for this study were all established using an approximately constant silviculture, well before the advances in establishment techniques and tree breeding of the mid-1970s. None of the plots had received any fertiliser application. To reduce the influence of silvicultural treatment, only unthinned plots were selected. The stocking of the plots selected ranged at first measurement from 1208 to 1863 (average 1603) stems ha^{-1}.

On every occasion of measurement, diameter was measured overbark at 1.3 m and all dead trees were recorded, so total production basal area is available for this modelling exercise. At the time of any volume measurement or plot thinning, the diameter, height and volume of all thinned or dead trees were measured. This enables the total production basal area and volume to be calculated.

Upper stand height or predominant height (*H*) is defined in South Australia as the mean height of the tallest 75 trees ha^{-1} with the restriction that the number of trees selected from each quarter of the plot must be as close to the same as possible. Given the narrow range of plot size, this provides a consistent number of trees measured. In the early years some plots had predominant height estimated by the person who had climbed one of the sample trees, the estimate being based on the known heights of the various sample trees and other selected trees that had been measured for tree height. The methodology evolved until the late 1950s when the current procedure was formalised. When the current strict definition was adopted, the late A. Keeves (*pers. comm.*) analysed the various techniques that had been used and concluded that there were no material differences in the various estimates of predominant height. This analysis was based on the sample plots where all trees were climbed for volume and height — the plots that were the basis of his thesis (Keeves 1961).

Volume estimates in the PSPs have never been determined from simple two-variable tree volume equations or taper equations, but always obtained from measured samples. In 1935 the method was to select sample trees of approximately the quadratic mean diameter and to climb those, to obtain data according to a standard sectional technique, measuring over and underbark diameters based on 10-foot sections to a 4-inch top diameter under bark. Jolly pioneered the use of the linear relationship between tree volume and basal area (Jolly 1950; Keeves 1961; Lewis *et al.* 1976) using a fixed starting point for the line and ensuring that the volume line passed through the sample mean. Soon this approach was enhanced by the use of 'three group sampling' with about three trees sampled in each group, above, below and approximately at the mean tree diameter. The volume line adopted was generally the average of the lines from the mean group to the upper and lower groups. In 1961 Keeves demonstrated that the relationship was linear, one of the first uses of modern computing in forest mensuration, and from that time samples were selected at random across the diameter distribution and the volume line was determined by linear regression analysis. The conversion from imperial to metric was a soft conversion, with 10-foot sections becoming 3 m and the top diameter limit underbark was changed from 4 inches to 10 cm. In the late 1950s Lewis pioneered the development of a four-way tree volume equation, the regional volume table (RVT), based on underbark diameter — at 5 feet and either 15 feet or 25 feet — and tree height (Lewis and McIntyre 1959) which was converted to metric by Leech (Lewis *et al.* 1973). If the RVT was used for sample tree measurement then more samples were taken so that any imprecision caused by the use of a volume table was offset by a reduction in sampling error. Stand volume measurements are as consistent as can be found in any forestry enterprise. Any possible error incurred by using the RVT can be assessed by evaluating sectionally measured climbed samples, or by sectional measurements on felled trees.

All trees that die or are thinned are measured by the sectional method, enabling total production volume to a 10-cm top diameter underbark to be calculated.

Standing volume is expensive to measure and so measurements were restricted to measurement at each thinning and generally half way between thinning; for unthinned plots, measurements were either at about a 5-y interval or when any paired thinned plot was measured. Basal area was commonly measured more frequently.

For this study the data base was restricted in an attempt to ensure that the analysis was not confounded by different numbers of measurements, nor by thinning, fertiliser application, different establishment techniques, or effects of tree breeding. Only plots in the lower south-east region of South Australia were selected, narrowing the range of annual rainfall. Further, the few plots of very poor growth were not considered, primarily because the current forest estate has consistently higher growth rates. One plot with a very high growth rate was not considered because the growth rate and pattern of growth are believed to be unusual and atypical because a great deal of extra water and nutrients wash on to the site from land uphill.

All 12 unthinned plots selected had at least nine measurements for total production volume, total production basal area and predominant height, a total of 129 measurements. The data base is limited but it is orthogonal, with equal numbers of measurements of all variables. This restriction would not normally be used if developing predictive models for practical use, but was imposed here to avoid any concern that a varying number of data might affect the comparisons. The age 10 y values for the three variables were estimated by interpolation — or in the case of predominant height for a few plots by extrapolation using existing measurements, confirmed by the Lewis Yield Table (Lewis *et al.* 1976), with a maximum extrapolation of 2 y. Figures 1–3 depict the data.

Inspection of the three graphs demonstrates the long-term trend nature of the data. Inspection also shows that there is a greater variation about a general curve of best fit in the trends for predominant height for each plot. This was just as expected. The volume trends have a little more variation than the basal area trend. The age 46 y measurement of volume for one of the plots of high site potential seems to be higher than expected — this is possibly due to there being fewer samples measured at that age, it being more common to measure more samples at any age 50 y measurement. The graphs indicate, however, that it should be possible to develop satisfactory models.

The data base does not represent the south-east of South Australia and, in that only unthinned plots are used, it does not represent current silvicultural practice. For many years almost all the plantation estate has been commercially thinned at some stage during a rotation. Further, since about 1976, plantation establishment practices have advanced and these changes are known to have changed plot growth. Any models presented in this paper should therefore not be used in practice. The data base is, however, most appropriate for discriminating between modelling approaches as it is orthogonal and includes a limited range of alternative silvicultural practices.

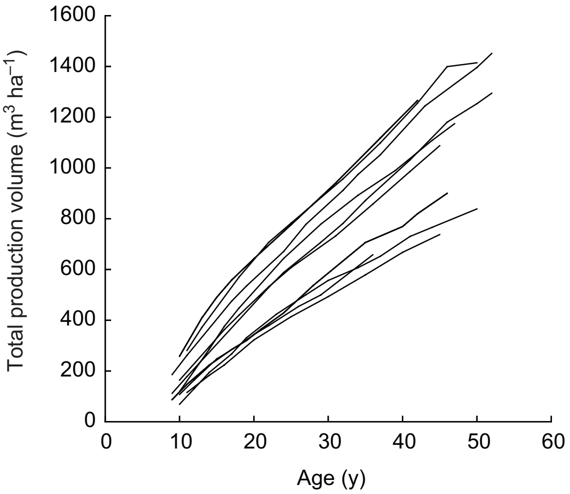

Figure 1. Volume for each plot

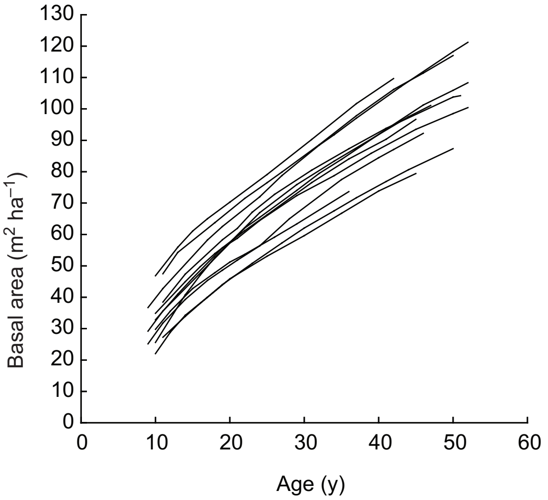

Figure 2. Basal area for each plot

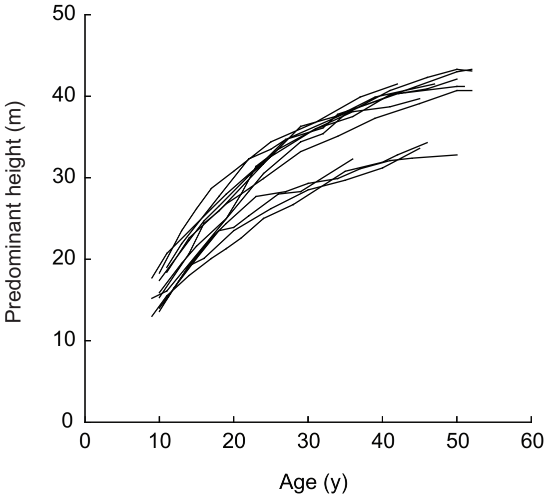

Figure 3. Predominant height for each plot

Model development

Two sets of models are required: models to predict volume from basal area and predominant height, and growth models to predict volume, basal area and predominant height growth over time.

A volume prediction model based on all measurements

Using 129 observations from the 12 plots, the simple combined variable model was calculated, both weighted by $(1/(B.H))$ and unweighted. As the standard errors for the weighted model were lower, this model was preferred. An added constant was found to be not significant. Residual plots showed the model to be satisfactory and the model was:

$$\begin{array}{l} V = \;0.2895\,BH \\ \qquad\;(0.0014)\,, \end{array}$$

where the standard error of the parameter estimate is shown in brackets below the estimated parameter. When the power parameters and the constant were allowed to fluctuate, the model was not significantly improved although the parameters were approximately as expected. Even setting ($b_1 + b_2 = 2$) to conform with dimensional analysis did not assist. When age was added as a possible explanatory variable it was not statistically significant.

A volume prediction model based only on the first measurement

The first measurement parallels the inventory measurement and so two alternative volume predictors can be evaluated — the model above based on all measurements, and a separate model based only on the first measurement in each plot. The equivalent model was:

$$\begin{array}{l} V = \;0.2780BH \\ \qquad\;(0.0072)\,. \end{array}$$

Not surprisingly, the standard error of the parameter estimate was higher with only 12 observations. Other structures were also evaluated but none was significantly better.

A volume growth model

Various first-stage models were developed for each of the 12 plots. The best structure was the constrained Mitscherlich model — Leech (1978) also found this to be the most appropriate model. For all plots, both parameters were significantly different from zero, and the structure of the model and residual analysis suggested that the model was appropriate.

The best second-stage model for p was:

$$\begin{array}{l} p = \;0.01651 \\ \qquad\;(0.00752) \end{array}$$

and adding in various V_{10} terms did not improve the fit. This structure does not parallel the results of the Leech (1978) analyses, but the difference is probably caused by the exclusion of the plots on sites of lower potential and thinned plots, and there being far fewer plots used.

The best-second stage model for the $a0$, the age at which volume growth commences, was as developed by Leech (1978):

$$\begin{array}{l} a0 = 10\exp(-0.003912\;V_{10}) \\ \qquad\qquad\qquad\quad(0.000244)\,. \end{array}$$

Linear and quadratic models were also evaluated, but they were poorer predictors than this constrained nonlinear model that makes structural sense in that when ($V_{10} = 0$) the parameter ($a0 = 0$).

The volume growth model was therefore:

$$V_A = V_{10}\left[\frac{\begin{array}{r} 1-\exp(-0.01651 \\ \times(A-(10\exp(-0.003912\;V_{10})))) \end{array}}{\begin{array}{r} 1-\exp(-0.01651 \\ \times(10-(10\exp(-0.003912\;V_{10})))) \end{array}}\right]$$

and this model has only two parameters.

A basal area growth model

When the three-parameter first-stage models were developed for each of the 12 plots, the model failed to converge for nine plots and for two others provided a curve of rather unusual shape, indicating that there were too few data to develop three-parameter models for each plot. Allowing only the parameter p to fluctuate provided structurally unusual and inconsistent models for nine of the 12 plots, so the single-parameter model form was also rejected as being structurally inadequate. The two two-parameter models, one with p and $a0$, the other with p and m, were then evaluated. Allowing the parameter m to fluctuate was superior to allowing $a0$ to fluctuate for ten of the 12 plots, and so this model structure was accepted as the best first-stage model structure.

The best second-stage model for p was:

$$\begin{array}{lcccc} p = & 0.30775 & - & 0.08120 & \ln(B_{10}) \\ & (0.09940) & & (0.02863) & \end{array}$$

with the replacement of $\ln(B_{10})$ by B_{10} providing a poorer fit. The addition of the quadratic term to the linear term was not significant.

The best second-stage model for m was:

$$\begin{array}{lcccc} m = & 0.5469 & - & 0.0005885 & (B_{10})^2 \\ & (0.0971) & & (0.0000783)\,. & \end{array}$$

The simple linear model was less efficient statistically (greater error sum of squares) and provided unreasonable predictions for very low values of B_{10} outside the data range. If both linear and quadratic terms were tried together, neither was significant. Evaluating the nonlinear model assuming the quadratic term was allowed to fluctuate provided a power term not significantly different from 2.0. Various reciprocal and logarithmic transform models were also evaluated, but the simple two-parameter model was the most efficient, and both parameters were significantly different from zero.

The combined model is the four-parameter model:

$$B_A = B_{10}\left[\frac{1-\exp(-p(1-m)(A))}{1-\exp(-p(1-m)(10))}\right]^{1/(1-m)}$$

with

$$p = 0.30775 - 0.08120\ln(B_{10})$$
$$m = 0.5469 - 0.0005885(B_{10})^2 .$$

A predominant height growth model

Like the basal area models, the three-parameter first-stage models were unsatisfactory for predominant height. Allowing only the parameter p to fluctuate provided structurally reasonable models for all plots.

An inspection of the parameter estimates showed a very poor relationship with site potential, H_{10}. The best second stage for p was the simple constrained model:

$$\begin{aligned} p = \ & 0.002731 H_{10} \\ & (0.000102) . \end{aligned}$$

The addition of a constant was not significant and this model had about half the error sum of squares of the equally simple one-parameter model with p set to a constant. The reason for this simpler structure was consistent with the belief that height models are less precise than volume models, the basic tenet underlying the decision by N.W. Jolly and N.B. Lewis to model volume growth directly.

The predominant height growth model is therefore:

$$H_A = H_{10}\left[\frac{1 - \exp(-0.002731\, H_{10}\, A)}{1 - \exp(-0.002731\, H_{10}\, 10)}\right].$$

Evaluating the two-parameter models at a plot level, allowing p and $a0$ to fluctuate, and allowing p and m to fluctuate, provided some challenges. When $a0$ was allowed to fluctuate, for two of the 12 plots the estimated value of $a0$ was negative, which was structurally unsatisfactory, and the estimated values were significantly different from zero for seven of the plots, again unsatisfactory, as predominant height should approximate zero (or about 0.2 m) at age zero. The models allowing m to fluctuate provided reasonable models, but overall were not markedly superior predictors for each plot. When the second-stage models were evaluated, the estimated values of both $a0$ and m were both not significantly different from zero and there was no obvious relationship with H_{10}. The two-parameter first-stage models were rejected.

Model development conclusions

The models developed were quite simple structurally. Simplicity is generally considered a good attribute in modelling.

For the stand volume models, the simple combined variable form, developed weighted, was the most efficient model.

For the growth models, the volume model has two parameters, the basal area model four, and the predominant height model one. Part of the reason for this simplicity is the constrained model structure that was also shown to be efficient elsewhere (Leech 1978), but part could be because only a limited number of plots were used in the analysis.

Model evaluation

One simple way of validating the growth model is to compare the three growth models with the Lewis Yield Tables (Lewis *et al.* 1976). Figures 4–6 show the comparisons. It is of interest that the predominant height–age graphs show little divergence as age increases.

The figures demonstrate that the minimum data base used for this analysis can provide models that are quite compatible with the yield curves developed on data from considerably more thinned and unthinned plots. The growth models would seem to be quite satisfactory.

The volume growth model tends to provide lower predictions at later ages than the Lewis Yield Table. This was completely expected, as the data used for this analysis were from unthinned plots whereas the Lewis Yield Table was based on both thinned and unthinned plots. This trend was also found by Leech (1978). The basal area growth models fit reasonably well up until age 40 y. Inspection of the data shows that this trend is correct for the data set used. The predominant height model appears to be less consistent, but the differences are due in part to the plots selected.

Evaluation of predictions

The models can be used to evaluate the two alternative methods of predicting growth. It must be stressed that the data are not independent and that the test is based on the developmental data. It should also be stressed that the models are based on only a small subset of the data now available, as this minimised any possible silvicultural effects. Also, the data base had been reduced to a perfectly orthogonal data set so as to eliminate any effects of variations in the amount of data on the analysis. Every effort was made to make the comparison as fair as possible.

The test was to compare the ability of the two alternatives to take basal area and predominant height information at the first of the available measurements, and to use these two pieces of data to predict volume at each subsequent measurement. To eliminate the effect of increment period, it was decided to compare the periodic annual increment, as modelled from the first measurement to each of the subsequent measurements, with the actual increment observed for that plot measurement. There were 117 measurements from 12 plots. The measurements are not strictly independent, but this was not considered vital as the same conditions applied to both alternative analyses.

For alternative 1, the basal area and predominant height values at the first measurement were used to determine the equivalent values at age 10 y. This required the development of a binary search style algorithm. The site potential values (B_{10} and H_{10}) that were derived were then used to predict the basal area and predominant height at each subsequent measurement. Volume was obtained using the stand volume equation. The predicted volume increment was compared against the actual volume increment and summarised. The mean and standard deviation for the error in the prediction of periodic annual increment was -2.18 ± 4.49 m^3 ha^{-1}.

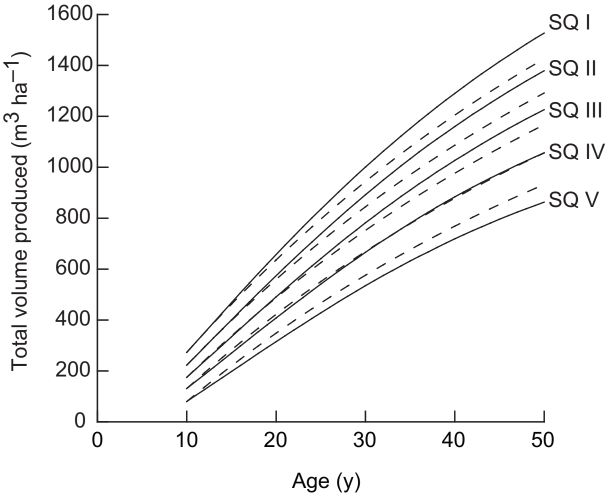

Figure 4. Lewis yield table for total production volume (solid line) and volume growth model (dotted line); SQ = site quality

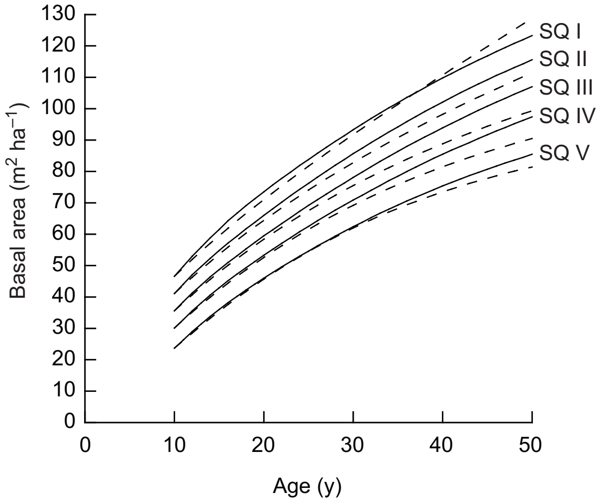

Figure 5. Graph of Lewis yield table for total production basal area (solid line) and basal area growth model (dotted line); SQ = site quality

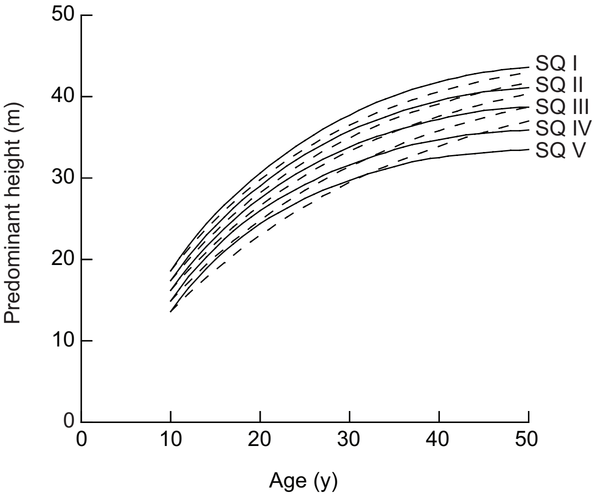

Figure 6. Graph of Lewis yield table for predominant height; (solid line) and predominant height growth model (dotted line)

For alternative 2, the basal area and predominant height values at the first measurement were used to determine the volume at that age. This was then used with the volume growth model to predict the volume at age 10 y using a binary search style algorithm. This site potential value derived was then used to predict the volume at each subsequent measurement. The predicted volume increment was compared against the actual volume increment and summarised. The mean and standard deviation for the error in the prediction of periodic annual increment was –1.05 ± 4.28 m^3 ha^{-1}.

The comparison shows that alternative 2, predicting volume growth, had a lower mean error and, more importantly, a lower standard deviation for those errors, suggesting that predicting volume growth is marginally superior to predicting basal area and upper stand height growth. It was also based on a two-parameter volume model compared with a four-parameter basal area and a single-parameter upper stand height model.

It should be noted that neither mean error is significantly different from zero and that both techniques provide quite reasonable predictions.

The length of the projection period averaged 19.2 y and ranged from 2 to 42 y. The average periodic increment was 529 m^3 ha^{-1}, and the average periodic annual increment was 29.3 m^3 ha^{-1}.

Discussion and conclusions

Given that the data base was completely orthogonal and that there were no other differences in the approaches used apart from the two alternative prediction philosophies, the conclusion is simple: it is marginally better in a statistically efficient sense to predict volume growth than it is to predict the growth of basal area and upper stand height. Both techniques, however, can provide good predictions.

Given the relatively narrow data base used the models presented should not be used in practice for predicting future volume yields.

The decision to use a volume growth prediction approach commenced by N.W. Jolly, then carried on by N.B. Lewis, and now embodied in the Yield Regulation System used by ForestrySA, is shown to have been soundly based.

It is recognised that this outcome may be influenced by the nature of height growth on the wind-swept coastal plains, and that similar analyses elsewhere may possibly provide contrary findings.

General knowledge of other data bases and approaches used in forest planning models throughout Australasia and elsewhere suggests that few organisations are in a position to replicate this analysis. They rarely, if ever, have the long-term data of volume, basal area and upper stand height measured to consistent standards and from stands subjected to a narrow range of silviculture. This leads one to question whether the modelling approach adopted by other organisations is based on sound reasoning and analysis or is based simply on what is possible and what has been done before.

Determining product volumes

This analysis provides predictions of total volume, whereas increasingly forest managers and planners require the prediction of volumes by different product categories.

This argument has been raised in support of the use of basal area and height growth models, rather than volume growth models, in that the former facilitate the use of taper equations which can enable volumes to be determined for products that differ in both size and quality.

Taper functions can be developed only rarely for local areas — it is common to develop one generic taper equation and then to assume that this holds over the whole forest estate and at all ages. The application error that is likely to be introduced by this assumption can be quite large, although overall the predictions may well be unbiased. As growers increasingly want to have unbiased estimates at a logging coupe level, the need to develop local taper equations is becoming increasingly important.

Importance sampling (Gregoire *at al.* 1986; Wiant *et al.* 1989; Wood and Wiant 1992; Leech 1996) provides one way of providing unbiased estimates of volume at a logging coupe level. In this approach an upper stem diameter is measured and used to correct the volumes based on a proxy taper function. Experience suggests that the proxy taper function does not have to be very good for the technique to be very effective in predicting total volume.

If the inventory is enhanced to allow the measurement of an upper stem diameter on a sample of trees at inventory, then the sample could be used to provide a correction to a more-generally appropriate taper function that would therefore provide a locally unbiased taper function. This would overcome one of the main difficulties with the common application of the MARVL approach, and would enable unbiased estimates of volume at inventory without the need to develop tree or stand volume equations as has been done in this analysis.

Prediction of future product volumes then depends on whether it is better to accept the flaws in assuming that a standard taper function holds at all ages and over all areas, and also accepting the marginally poorer predictions that the use of basal area and upper stand height growth models occasions, or whether it is superior to model volume growth and to develop product prediction equations based on age and other stand and site variables. This would require a considerably more complex analysis and the availability of suitable data. The analysis in this paper, however, does demonstrate the marginal superiority of growing volume to growing both basal area and upper stand height.

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